

Mathematical Statistics

Estadística Matemática Extremadura

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GOBIERNO DE EXTREMADURA

Consejería de Empleo, Empresa e Innovación



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Overview

- 1 Lines of Research
- 2 Markov kernels
- 3 Nonparametric statistics
- 4 Flexible modelling

Lines of Research

- 1 Markov Kernels**
Agustín García Nogales
- 2 Nonparametric Statistics**
José Enrique Chacón Durán: Tarn Duong (Sorbonne, France), Carlos Tenreiro (Coimbra, Portugal)
- 3 Flexible Modelling of Linear and Directional Data**
Arthur Pewsey: M.C. Jones (Open, UK), Toshihiro Abe (Nanzen, Japan), Shogo Kato (ISM, Japan)
- 4 Applications of Statistics in Medical Research**
Jesús Montanero Fernández

Markov kernels

 Agustín García Nogales

Markov kernels, also referred to as **stochastic kernels** or **transition probabilities**, are fundamental to Probability Theory and Mathematical Statistics because, in one sense, any Markov kernel is a **conditional distribution**.

Examples of Markov kernels include:

- 1 **Transition matrices** of Markov chains,
- 2 **Posterior distributions** in Bayesian analysis,
- 3 **Decision rules** in decision theory.

A **Markov kernel** can also be considered to be a generalization of the concepts of **random variable** and **sub- σ algebra**.

References

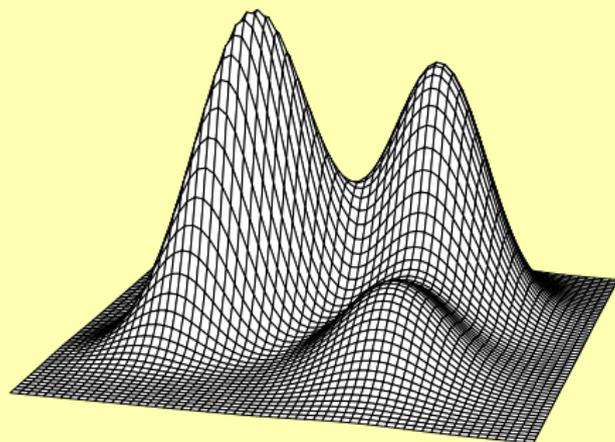
In recent years, we have extended the concepts of **completeness**, **ancillarity**, **independence** and **regular conditional probability** to Markov kernels.

Nogales, A.G. (2013) On independence of Markov kernels and a generalization of two theorems of Basu. **Journal of Statistical Planning & Inference**, **143**, 603–610.

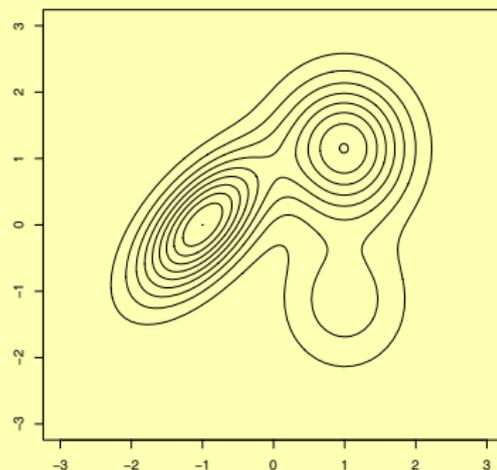
Nogales, A.G. (2013) The existence of regular conditional probabilities for Markov kernels. **Statistics & Probability Letters**, **83**, 891–897.

Nonparametric Statistics José Enrique Chacón Durán

The primary focus of this line of research is **nonparametric density estimation** for data distributed on \mathbb{R} or \mathbb{R}^p .



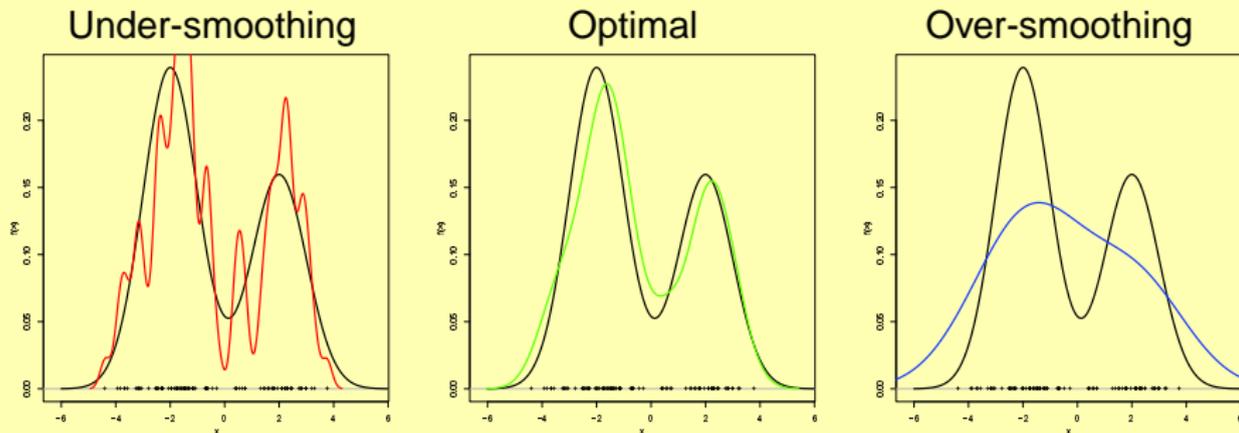
Perspective plot



Contour plot

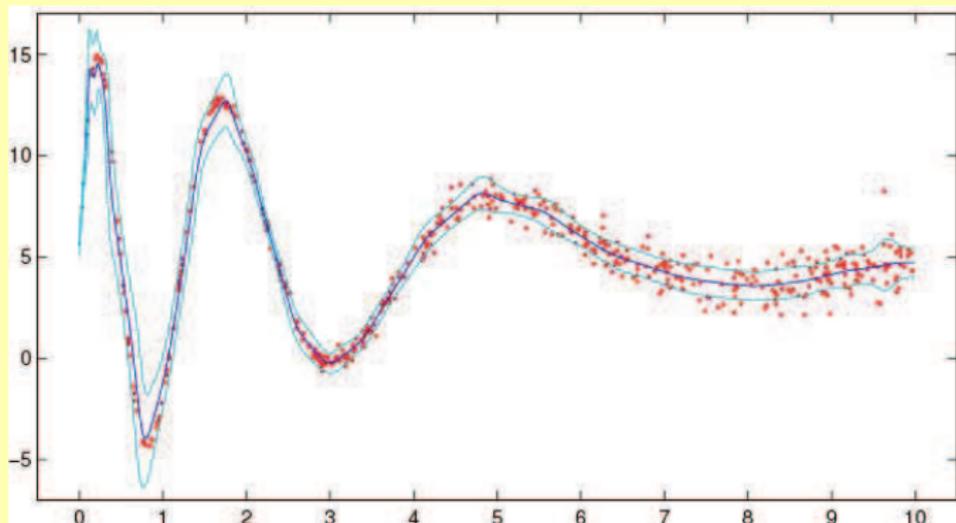
Nonparametric Statistics

The main approach to **nonparametric density estimation** considered is that based on **kernel smoothing** and the related problem of **bandwidth selection**.



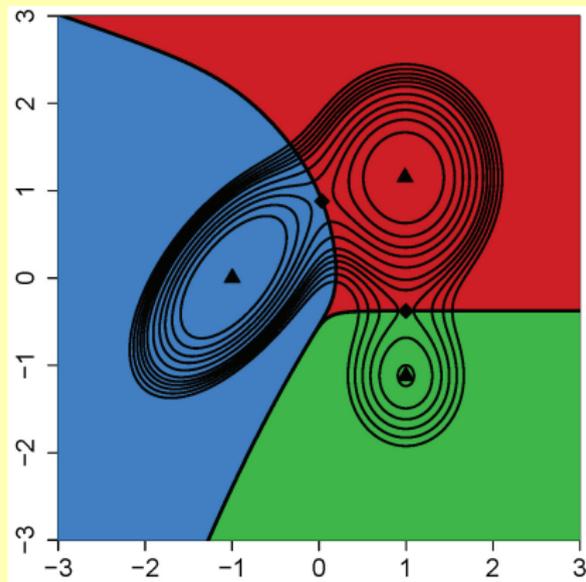
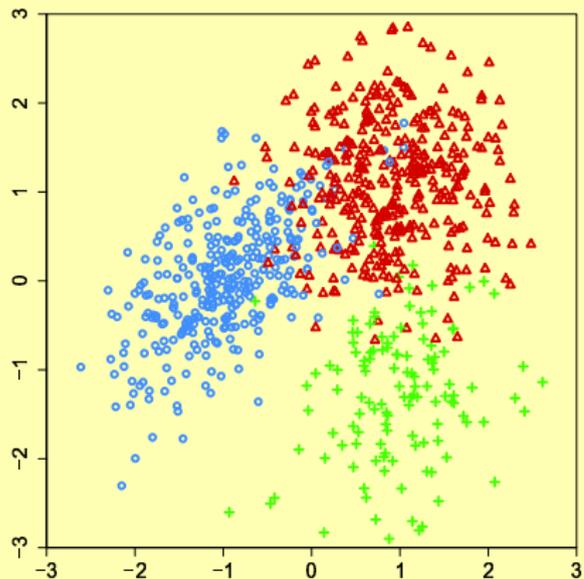
Nonparametric Statistics

A related problem is **nonparametric regression**.



Nonparametric Statistics

A further related problem is **cluster analysis**.



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Flexible modelling

Arthur Pewsey

The focus for this line of research is the **flexible** parametric modelling of **linear** data, distributed on (subsets of) \mathbb{R} or \mathbb{R}^p , and **directional** data, distributed on the **unit circle**, **torus**, **cylinder**, **sphere**, or **multivariate extensions** of such supports.

By **flexible** we mean that the models should be able to describe features such as **varying degrees** of **asymmetry**, **kurtosis**, **shape** and **multimodality** often manifested by **real data**. For data on \mathbb{R} , this usually means that the **number of parameters** involved is 4 or more.

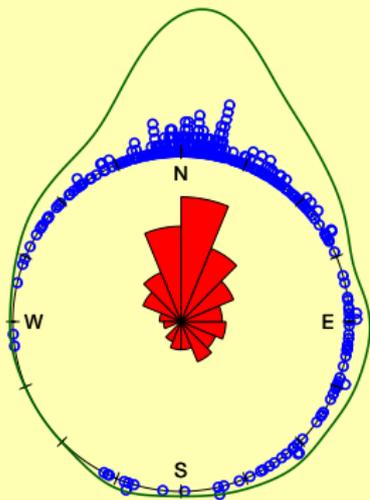
Directional data

Directional data arise, for example, when we measure:

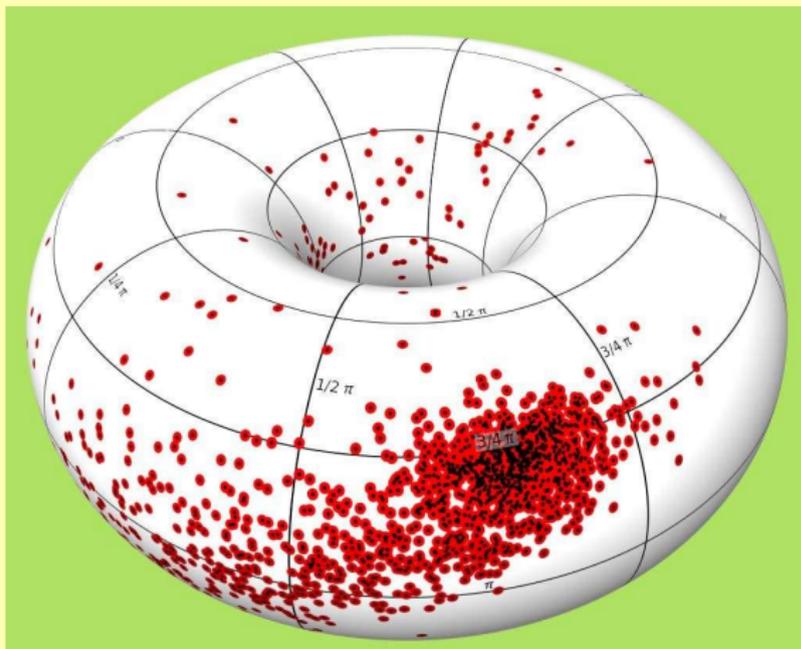
- 1** **Wind direction** (an angle or direction of a unit vector in 2D: **circular**),
- 2** **Ozone concentration** and **wind direction** (a real variable and an angle: **cylindrical**),
- 3** Pairs of **dihedral angles** between the **amino acids** making up a **protein** (two angles: **toroidal**),
- 4** **Locations** of **impacting meteorites** (direction of a unit vector in 3D: **spherical**).

Directional data on compact manifolds

Circular



Toroidal



References

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- Jones, M.C., Pewsey, A. & Kato, S. (2015) On a class of circular copulas for circular distributions. *Annals of the Institute of Statistical Mathematics*, **67**, 843-862.