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# TRANSITIVITY IN SPACES OF VECTOR-VALUED FUNCTIONS

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*Dedicated to the memory of Antonio Aizpuru Tomás (1954–2008)*

*Abstract* We exhibit a real Banach space $M$ such that $C(K, M)$ is almost transitive if $K$ is the Cantor set, the growth of the integers in its Stone–Čech compactification or the maximal ideal space of $L^\infty$. For finite $K$, the space $C(K, M) = M^{|K|}$ is even transitive.

*Keywords:* Banach space; isometry; transitivity; vector-valued function

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## 1. Introduction

A Banach space $X$ is almost transitive when the orbits of the isometry group are dense in the unit sphere: given $x, y \in X$ with $\|x\| = \|y\| = 1$ and $\varepsilon > 0$, there is a (linear, surjective) isometry $T : X \to X$ such that $\|y - Tx\| \leqslant \varepsilon$. When this can be achieved even for $\varepsilon = 0$, the isometry group acts transitively on the unit sphere and $X$ is said to be transitive.

Transitivity problems have spurred a moderate interest in Banach space theory since its inception. The most important open problem in the area is whether Hilbert space is the sole transitive separable Banach space. This seemingly untractable problem was posed by Mazur in the 1930s and then recorded by Banach in [**3**]. We refer the reader to [**23**] and the survey papers [**4**, **5**] for information on the topic.

Transitivity in spaces of continuous functions also attracted attention. Recently, Rambla [**22**] and, independently, Kawamura [**16**] found a locally compact space $L$ with $C_0^{\mathbb{C}}(L)$ almost transitive in its natural supremum norm; needless to say, $L$ is not a singleton. This solved in a somewhat unexpected way a question raised by Wood in [**25**].

In this paper we present examples of Banach spaces of vector-valued functions that are transitive or almost transitive. More precisely, we exhibit a real Banach space $M$ such that $C(K, M)$ is almost transitive if $K$ is the Cantor set, the growth of the integers in its Stone–Čech compactification or the maximal ideal space of $L^\infty$. For finite $K$, the space $C(K, M) = M^{|K|}$ is even transitive. This settles a problem previously considered in [**13**, § 5] and posed explicitly in [**1**, Question 6.5].

At the end of the paper we will use a variation on Kawamura–Rambla example to answer a question of Pestov on amenability of almost-transitive groups of isometries.

### 1.1. Notation

Given a locally compact (Hausdorff) space $L$ and a Banach space $X$, we write $C_0(L,X)$ for the Banach space of all continuous functions $f : L \to X$ vanishing at infinity, equipped with the norm $\|f\| = \sup_{t\in L} \|f(t)\|$. If $L$ is compact, the subscript will be omitted. We use the identification of $C_0(L,X)$ with the injective tensor product $C_0(L)\check{\otimes}_\varepsilon X$ (see [**9**] for proper definitions) mainly for notational purposes; indeed, if $g \in C_0(L)$ and $x \in X$, then $g \otimes x : L \to X$ is given by $t \mapsto g(t)x$. The characteristic function of the set $B$ is denoted by $1_B$; the identity on $X$ is denoted by $I_X$.

We write $X \times Y$ for the product of the Banach spaces $X$ and $Y$ with the maximum norm $\|(x,y)\| = \|x\| \vee \|y\|$. The same linear space with the norm $(\|x\|^p + \|y\|^p)^{1/p}$ is denoted $X \oplus_p Y$. If $(X_n)$ is a sequence of Banach spaces, then $\ell^\infty(X_n) = \{(f_n) : f_n \in X_n$ for all $n$ and $\|(f_n)\|_\infty = \sup_n \|f_n\| < \infty\}$, with the obvious norm. We write $X \approx Y$ to indicate that $X$ is linearly isometric to $Y$, and $K \sim L$ means that $K$ and $L$ are homeomorphic. Finally, the group of homeomorphisms of $L$ is denoted $H(L)$.

## 2. Spaces of vector-valued functions

The following remark leads to the sought-after counter-examples.

**Lemma 2.1.** *If $X$ is an almost-transitive space isometric to $X\times X$ and $K$ is a compact space whose topology has a base $\mathfrak{B}$ (necessarily of clopen sets) such that $B \sim K \sim B^{\mathrm{c}}$ for every $B \in \mathfrak{B}$, then $C(K,X)$ is almost transitive.*

**Proof.** Let us say that $f : K \to X$ is very simple if there is a decomposition $K = B_1 \oplus \cdots \oplus B_m$, with $B_k \sim K$ such that $f = \sum_{k=1}^m 1_{B_k} \otimes x_k$. The hypothesis on $K$ and the Stone–Weierstrass Theorem imply that very simple functions are dense in $C(K)$. Therefore, very simple functions are dense in $C(K,X) = C(K)\check{\otimes}_\varepsilon X$.

Suppose now we are given a decomposition $K = B_1 \oplus \cdots \oplus B_m$, with $B_k \sim K$. Writing $K \oplus \overset{m}{\cdots} \oplus K = K \times \{1,\dots,m\}$ and $X \times \overset{m}{\cdots} \times X = X^m$ we have a homeomorphism $\varphi : K \times \{1,\dots,m\} \to K$ and an isometry

$$T_\varphi : C(K,X) \to C(K \times \{1,\dots,m\}, X) = C(K, X^m)$$

taking $f \in C(K,X)$ into the function $x \mapsto (f(\varphi(x,k)))_{k=1}^m$. Notice that for $f = \sum_{k=1}^m 1_{B_k} \otimes x_k$ one has $T_\varphi(f) = 1_K \otimes (x_1,\dots,x_m)$.

We complete the proof by showing that the orbit of every (normalized) constant function is dense in the unit sphere of $C(K,X)$. Indeed, let $g = 1_K \otimes y$ be any constant function with $\|y\| = 1$ and let $f = \sum_{k=1}^m 1_{B_k} \otimes x_k$ be a very simple function of norm 1: $\|x_1\| \vee \cdots \vee \|x_m\| = 1$. Take $\varphi$ as before, so that $T_\varphi f = 1_K \otimes (x_1,\dots,x_m)$ and $T_\varphi g = 1_K \otimes (y,\dots,y)$. As $X^m$ is almost transitive (it is isometric to $X$), given $\varepsilon > 0$, there is an isometry $S$ of $X^m$ such that $\|(x_1,\dots,x_m) - S(y,\dots,y)\| < \varepsilon$. Now, if $I_{C(K)}$ denotes the identity on $C(K)$, we can ‘extend’ $S$ to an isometry on

$C(K,X^m) = C(K)\check{\otimes}_\varepsilon X^m$ just by taking $I_{C(K)} \otimes S$, and it is pretty obvious that $\|f - Tg\| < \varepsilon$, where $T = T_\varphi^{-1} \circ (I_{C(K)} \otimes S) \circ T_\varphi$. □

The following are some compact spaces fulfilling the hypothesis of the lemma.

- The Cantor set $\Delta = \{0,1\}^{\mathbb{N}}$.
- The growth of the integers in its Stone–Čech compactification: $\mathbb{N}^* = \beta\mathbb{N}\setminus\mathbb{N}$. Recall that $C(\mathbb{N}^*) = \ell^\infty/c_0$.
- The maximal ideal space $\mathfrak{M}$ of $L^\infty(0,1)$, so that $C(\mathfrak{M}) = L^\infty(0,1)$.

Some explanations are in order. That these compacta have the required property follows from the fact that they are totally disconnected (hence they have bases of clopen sets) and, moreover, every non-empty clopen set is homeomorphic to the whole space. For the Cantor set this follows from Hausdorff's [**15**] characterization of $\Delta$ as the only totally disconnected, perfect, compact metric space. For $\mathbb{N}^*$ just follow the indications given in [**10**, Problem 6S, p. 98]. As for the maximal ideal space of $L^\infty(0,1)$, suppose that $\mathfrak{N}$ is a non-empty clopen set of $\mathfrak{M}$. Then $1_{\mathfrak{N}}$ is an idempotent of $C(\mathfrak{M})$ that corresponds to a unique idempotent in $L^\infty(0,1)$. Each (non-zero) idempotent in the latter algebra has the form $1_A$, where $A$ is a Borel subset of positive measure of $(0,1)$. We have $C(\mathfrak{N}) = 1_{\mathfrak{N}} \cdot C(\mathfrak{M}) \simeq 1_A \cdot L^\infty(0,1) = L^\infty(A) \simeq L^\infty(0,1) \simeq C(\mathfrak{M})$, where '$\simeq$' means that the corresponding Banach algebras are isometrically isomorphic, and so $\mathfrak{N} \sim \mathfrak{M}$.

It is perhaps worth noting that the relevant property of $K$ is trivially stable by products; thus, we can combine the above examples to get new ones.

The following example provides the target space we need.

**Example 2.2.** A transitive Banach space $M$ isometric to $M \times M$.

**Proof.** The following construction was introduced in [**6**, Lemma 3.2] with a different purpose. Let $p(n)$ be a sequence of real numbers tending to $\infty$ and consider the spaces $L^{p(n)} = L^{p(n)}(0,1)$. Let $U$ be a non-trivial ultrafilter on the integers. Then $M$ is the ultraproduct of the family $(L^{p(n)})$ along $U$, that is,

$$M = [L^{p(n)}]_U = \ell^\infty(L^{p(n)})/N_U,$$

where $N_U$ is the subspace of those $(f_n) \in \ell^\infty(L^{p(n)})$ such that

$$\lim_{U(n)} \|f_n\|_{p(n)} = 0.$$

We refer the reader to [**24**] for information on ultraproducts. Here we only recall that the norm in an ultraproduct (which is defined as a quotient norm) can be computed as

$$\|[(f_n)]\|_U = \lim_{U(n)} \|f_n\|,$$

where $[(f_n)]$ denotes the class of $(f_n)$.

Of course $M$ could depend on $U$ and $p(n)$ but we do not need to be so specific. The transitivity of $M$ follows from the almost transitivity of the spaces $L^p$ for $p < \infty$ (see [**13**, Theorem 1.2] or [**23**] and [**6**, Lemma 1.4]). Let us show that $M = M \times M$. Write $(0, 1) = A \oplus B$, where $A$ and $B$ are Borel sets of positive measure, or just take $A = (0, \frac{1}{2})$ and $B = [\frac{1}{2}, 1)$. One has

$$\begin{aligned} M &= [L^{p(n)}]_U \\ &= [L^{p(n)}(A) \oplus_{p(n)} L^{p(n)}(B)]_U \\ &= [L^{p(n)}(A)]_U \times [L^{p(n)}(B)]_U \\ &\approx M \times M. \end{aligned}$$

Here, the only non-trivial equality is the third one. But for $[(f_n)]$ in $M$ one has

$$\begin{aligned} \|[(f_n)]\|_U &= \lim_{U(n)} \|f_n\|_{p(n)} \\ &= \lim_{U(n)} \|1_A f_n + 1_B f_n\|_{p(n)} \\ &= \lim_{U(n)} (\|1_A f_n\|_{p(n)}^{p(n)} + \|1_B f_n\|_{p(n)}^{p(n)})^{1/p(n)} \\ &= \Big(\lim_{U(n)} \|1_A f_n\|_{p(n)}\Big) \bigvee \Big(\lim_{U(n)} \|1_B f_n\|_{p(n)}\Big), \end{aligned}$$

as desired. □

**Example 2.3.** Let $K$ be one of the spaces $\Delta$, $\mathbb{N}^*$ or $\mathfrak{M}$. Then $C(K, M)$ is almost transitive and so is $c_0(\mathbb{N}, M)$. □

I strongly suspect $C(\mathbb{N}^*, M)$ is actually transitive but I have been unable to find a proof. Of course, if $K$ is finite, then $C(K, M) = M^{|K|} \approx M$ is transitive, so $M$ 'itself' is a counter-example for the questions addressed in this note. The space $M$ could be isomorphic to a $C_0(L)$, but not to any $C_0(L, X)$ unless $L$ is finite or $X$ is finite dimensional. And this is so because every operator from $M$ to a separable Banach space is weakly compact (by the results in [**2**]), while $C_0(L, X)$ contains a complemented isomorph of $c_0$ as long as $L$ is infinite and $X$ is infinite dimensional: a well-known result by Cembranos [**8**]. In particular, $M$ is not isomorphic to $c_0(M)$. I do not know whether it is isometric to $\ell^\infty(\mathbb{N}, M)$ nor whether $\ell^\infty \check{\otimes}_\varepsilon M = C(\beta\mathbb{N}, M)$ is almost transitive. Notice that the former space is much bigger than the latter.

Moving to smaller spaces, we have the following.

**Example 2.4.** A separable Banach space $S$ such that $C(\Delta, S)$ is almost transitive.

**Proof.** We construct $S$ as a subspace of $M$ using an adaptation of the method in [**6**]. Let $M_0$ be any separable subspace of $M$. Then $C(\Delta, M_0)$ is a separable subspace of $C(\Delta, M)$ and we can find a countable group $G_0$ of isometries of $C(\Delta, M)$ such that for every $f, g \in C(\Delta, M_0)$ and each $\varepsilon > 0$ there is $T \in G_0$ such that $\|g - Tf\| < \varepsilon$. Now, let $X_0$ be the least $G_0$-invariant subspace of $C(\Delta, M)$ containing $C(\Delta, M_0)$.

Let $M_1$ be the least subspace of $M$ containing the range of all functions in $X_0$. Replacing $M_0$ by $M_1$ and continuing in this way, we get an increasing system $(M_n, G_n, X_n)$ such that

- $M_n$ is a separable subspace of $M$,
- $G_n$ is a countable subgroup of the isometry group of $C(\Delta, M)$,
- given $f, g \in C(\Delta, M_n)$ and $\varepsilon > 0$ there is $T \in G_n$ such that $\|g - Tf\| < \varepsilon$,
- $X_n$ is a separable $G_n$-invariant subspace of $C(\Delta, M)$,
- $M_{n+1}$ contains the range of every $f \in X_n$, that is, $X_n$ is a subspace of $C(\Delta, M_{n+1})$.

Finally, we set $S = \overline{\bigcup_n M_n}$. Clearly, $C(\Delta, S)$ is almost transitive. In fact, the restriction of the members in $\bigcup_n G_n$ acts almost transitively on the unit sphere of $C(\Delta, S)$. □

Notice that $S$ will be a sublattice of $M$ if we enlarge $M_{n+1}$ to be the least sublattice of $M$ containing the range of every function in $X_n$. Proceeding in this way, one guarantees that $S$ is isomorphic to $C(\Delta)$, by [**6**, Theorem 3.4]. In any case, $C(\Delta, S)$ is a separable almost-transitive space isometric to its square and having a good supply of $M$-projections.

One might consider Gurariĭ space [**14**] as the natural candidate for the target space in the lemma. Gurariĭ space $G$ was the first almost-transitive Lindenstrauss space appearing in nature and it has been widely studied. However, there is no non-trivial decomposition $G = E \times F$. Indeed, it is known [**18**] that the extreme points are weakly$^*$ dense in the unit ball of $G^*$. This cannot occur if $G^* = E^* \oplus_1 F^*$, with $E^*$ and $F^*$ non-trivial, as they are weakly$^*$ closed.

## 3. Almost-transitive groups of isometries

We take the opportunity here of answering a question in [**19**]. A topological group $G$ is said to be amenable if every continuous *affine* action of $G$ on a compact *convex set of a locally convex* space has a fixed point. By deleting all the italic words one obtains the notion of an extremely amenable group. It is proved in [**11**] that the isometry groups of the spaces $L^p$ are extremely amenable in the strong operator topology (SOT) for $1 \leqslant p < \infty$; the SOT in the space of operators $L(X, Y)$ is just the restriction of the product topology of $Y^X$. These results were generalized further in [**12**] and can be seen in [**20**, **21**].

This motivates the question of whether the isometry group of an almost-transitive Banach space must be extremely amenable in the SOT. Unfortunately, the answer is negative, as we shall see now. In fact, we shall show that such a group is not necessarily amenable.

**Lemma 3.1.** *Let $K$ be a homogeneous compact space and let $K_*$ be the locally compact space obtained by deleting one point in $K$. If $C_0(K_*)$ is almost-positive transitive, then $C(K)/\mathbb{R}$ is almost transitive under the quotient norm.*

**Proof.** Homogeneity means that, given $p, q \in K$, there is a homeomorphism $\phi$ of $K$ such that $q = \phi(p)$. This clearly implies that $K_*$ does not depend on the point removed and also that $K_*$ is a non-compact, locally compact space whose one-point compactification is $K$. Almost-positive transitivity means that, given $f, g \geqslant 0$ in the unit sphere of $C_0(K_*)$ and $\varepsilon > 0$, there is an isometry $T$ such that $\|g - Tf\| \leqslant \varepsilon$.

Next, observe that if $[f]$ denotes the class of $f \in C(K)$ in the quotient $C(K)/\mathbb{R}$, then

$$2\|[f]\| = \sup_{s,t \in P} |f(s) - f(t)| = \max f - \min f,$$

since the nearest constant to $f$ is $\frac{1}{2}(\max f - \min f)$. The isometries of $C(K)/\mathbb{R}$ were computed in [**7**]: they have the form $T[f] = [\pm f \circ \varphi]$, where $\varphi \in H(K)$ and $\pm$ is either 1 or $-1$.

Let us see that $C(K)/\mathbb{R}$ is almost transitive. Take $f, g \in C(K)$ such that $\|[f]\| = \|[g]\| = 1$ and $\varepsilon > 0$. Replacing $f$ and $g$ by $f - 1_K \min f$ and $g - 1_K \min g$, we may (and do) assume that $\min f = \min g = 0$ and $\max f = \max g = 2$. Pick $p$ such that $f(p) = 0$ and $q$ such that $g(q) = 0$. Let $\sigma \in H(K)$ such that $\sigma(p) = q$ and put $g' = g \circ \sigma$. Taking $K_* = K \setminus \{p\}$, we have $f, g' \in C_0(K_*)$, and since $\|f\| = \|g'\| = 2$ in $C_0(K_*)$ there is an isometry $T$ of $C_0(K_*)$ such that $\|g' - Tf\| \leqslant \varepsilon$. By the Banach–Stone Theorem we have $\|g' - f \circ \varphi\| \leqslant \varepsilon$, where $\varphi \in H(K_*)$ is the homeomorphism associated to $T$. Extending $\varphi$ to a homeomorphism of $P$ (which we still call $\varphi$), it is now clear that the self-map of $C(K)/\mathbb{R}$ given by $S[h] = [h \circ \varphi \circ \sigma^{-1}]$ is an isometry and also that $\|[g] - S[f]\| \leqslant \varepsilon$ in the norm of $C(K)/\mathbb{R}$. □

**Example 3.2.** An almost-transitive Banach space whose isometry group is not amenable in the SOT.

**Proof.** Kawamura and Rambla proved that if $P_*$ is obtained by deleting one point of the pseudo-arc $P$, then $C_0^{\mathbb{C}}(P_*)$ is almost transitive and so $C_0(P_*)$ is almost-positive transitive: actually, this is the crux, in both [**16**] and [**22**]. The pseudo-arc plays a major role in continuum theory; an account is given in [**17**]. As $P$ is homogeneous, we get from Lemma 3.1 that $C(P)/\mathbb{R}$ is almost transitive. But the isometry group $G$ of $C(P)/\mathbb{R}$ is algebraically isomorphic to $H(P) \times \{1, -1\}$, by the result in [**7**] we mentioned above. (The presence of the factor $\{\pm 1\}$ already prevents $G$ from being extremely amenable but we want to see that it is not amenable.) Let us represent the isometries of $C(P)/\mathbb{R}$ as pairs $T = (u, \varphi)$, where $u = \pm 1$ and $\varphi$ is a homeomorphism of $P$, so that $T[f] = [uf \circ \varphi]$. Then a net $T_\alpha = (u_\alpha, \varphi_\alpha)$ converges to $I_{C(P)/\mathbb{R}} = (1, I_P)$ in the SOT if and only if, for every $f \in C(P)$, one has

$$[f - u_\alpha f \circ \varphi_\alpha] \to 0$$

in the norm of $C(P)/\mathbb{R}$. It is easily seen that $u_\alpha = 1$ for $\alpha$ sufficiently large, so we in fact have $[f - f \circ \varphi_\alpha] \to 0$. On the other hand, $G = \pm H(P)$ acts by isometries both on $C(P)/\mathbb{R}$ and on $C(P)$, and so it carries two topologies, namely the $(C(P)/\mathbb{R})$-SOT and the $C(P)$-SOT. For reasons that will become clear later, we must show that these topologies agree on $H(P)$. This obviously implies that they also agree on $G$.

So let $(\varphi_\alpha)$ be a net converging to the identity in the $(C(P)/\mathbb{R})$-SOT; that is, for every $f \in C(P)$, one has

$$\|[f - f \circ \varphi_\alpha]\| \to 0.$$

We must show that, in fact, $\|f - f \circ \varphi_\alpha\| \to 0$. Fix $f \in C(P)$ and let $c_\alpha$ be the nearest constant to $f - f \circ \varphi_\alpha$. We have

$$\|f - f \circ \varphi_\alpha - c_\alpha\| = \|[f - f \circ \varphi_\alpha]\|.$$

Let us see that $f - f \circ \varphi_\alpha$ is pointwise null. This forces $(c_\alpha)$ to be null and

$$\|f - f \circ \varphi_\alpha\| \leqslant |c_\alpha| + \|[f - f \circ \varphi_\alpha]\| \to 0.$$

It clearly suffices to check that $\varphi_\alpha$ converges pointwise to $I_P$. To see this, take two different points $s, t \in P$ and consider the function (recall that $P$ is metrizable)

$$g(x) = \frac{d(x,t)}{d(x,t) + d(x,s)}, \quad x \in P.$$

Then $0 \leqslant g \leqslant 1$, and $g$ takes the value 0 only at $t$ and takes the value 1 only at $s$. As $\|[g - g \circ \varphi_\alpha]\| \to 0$ in $C(P)/\mathbb{R}$, we have, in particular, that

$$g(s) - g(\varphi_\alpha(s)) + g(\varphi_\alpha(t)) - g(t) \to 0$$

as $\alpha$ increases. Hence,

$$g(\varphi_\alpha(s)) - g(\varphi_\alpha(t)) \to 1,$$

which implies that $\varphi_\alpha(s) \to s$ and $\varphi_\alpha(t) \to t$.

This shows that the action of $G$ on $C(P)$ is continuous in the SOT. Now, we identify the dual of $C(P)$ as $M(P)$, the space of regular (signed) Borel measures on $P$ through the pairing $\langle \mu, f \rangle = \int_P f \, \mathrm{d}\mu$ and we put the corresponding weak* topology on $M(P)$. The 'dual action'

$$(G, \mathrm{SOT}) \times (M(P), w^*) \to (M(P), w^*)$$

given by $\langle (u,\varphi)\mu, f \rangle = \langle \mu, f \circ \varphi \rangle$ is continuous. Let $\Pi$ be the set of all probability measures in $M(P)$. This is a weak* compact set invariant under the given action. We are about to see that $G$ has no fixed point in $\Pi$. Indeed, suppose that $\mu \in M(P)$ is a fixed point, so that $\langle \mu, f \rangle = \langle \mu, f \circ \varphi \rangle$ for every $f \in C(P)$ and every $\varphi \in H(P)$. It is easily seen that there is a constant $c \geqslant 0$ such that $\langle \mu, g \rangle = c$ whenever $0 \leqslant g \leqslant 1$ attains the values 0 and 1 on $P$. This implies that $c = 0$ and so $\mu = 0$, which does not belong to $\Pi$. □

With a little more effort one can see that, for every countably incomplete ultrafilter $U$, the space $(C(P)/\mathbb{R})_U$ is transitive, yet its isometry group fails to be SOT-amenable, let alone extremely amenable.

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