Differential Geometry and its Applications 30 (2012) 216–221

Contents lists available at SciVerse ScienceDirect

Differential Geometry and its Applications

www.elsevier.com/locate/difgeo

Automorphisms of algebras of smooth functions and equivalent functions ☆

Félix Cabello Sánchez

Departamento de Matemáticas, Universidad de Extremadura, 06071-Badajoz, Spain

ARTICLE INFO

Article history:
Received 4 December 2011
Communicated by J. Stasheff

MSC:
58A05

Keywords:
Algebras of smooth functions
Automorphisms

ABSTRACT

We show that a linear map on $C^\infty(X)$ which agrees at every function with some automorphism (depending on the given function) is itself an automorphism.

1. Introduction and statement of the result

Let $C^\infty(X)$ the algebra of all real-valued smooth functions on a smooth manifold X. If $f \in C^\infty(X)$ and T is an automorphism of $C^\infty(X)$, then the function Tf has exactly the same geometrical properties as f.

A moment's reflection suffices to realize that, conversely, two smooth functions should be regarded as "equivalent" precisely when an automorphism of the corresponding algebra mapping one into the other exists.

The purpose of this short Note is to prove that the automorphisms of $C^\infty(X)$ are completely determined amongst the linear endomorphisms by the property of sending each function into an equivalent one. Let us record and label this properly.

Theorem. *Let Y and X be second-countable, finite-dimensional smooth manifolds with boundary. Let $T : C^\infty(Y) \to C^\infty(X)$ be a linear map having the property that, for each $f \in C^\infty(Y)$, there is an isomorphism of algebras $S : C^\infty(Y) \to C^\infty(X)$, possibly depending on f, such that $Tf = Sf$. Then T is an isomorphism of algebras.*

(We have used two different manifolds in the statement mainly for notational reasons.) We emphasize that S may depend on f.

To put the result in the proper setting, consider two linear spaces F and G (possibly with some additional structures) and let $L(F, G)$ denote the space of all linear maps from F to G. A subset $\mathfrak{S} \subset L(F, G)$ is said to be reflexive if it contains every $T \in L(F, G)$ which agrees at each $f \in F$ with some element of $\mathfrak{S}$, possibly depending on f.

Thus Theorem can be rephrased by saying that the group of automorphisms of $C^\infty(X)$ is reflexive.

☆ Research supported in part by MTM2010-20190 and by the program "Junta de Extremadura GR10113 IV Plan Regional I+D+i, Ayudas a Grupos de Investigación".

E-mail address: fcabello@unex.es.
URL: http://kolmogorov.unex.es/~fcabello.

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doi:10.1016/j.difgeo.2012.03.003

Reflexivity problems spurred a considerable interest in recent years. Most of the published works deal with derivations, automorphisms and isometries of Banach algebras. A good source on these matters and many related things is [10], specially Chapter 3.

By a manifold (in general with boundary) we understand a Hausdorff topological space X in which every point has a neighborhood homeomorphic to (an open set of) $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n\colon\ x_n \geqslant 0\}$ for some fixed n, called the dimension of X. The set of points of X having a neighborhood homeomorphic to $\mathbb{R}^n$ is called the interior of X and written $\operatorname{Int} X$. The complement of $\operatorname{Int} X$ in X is called the boundary of X and it is denoted by ∂X.

A smooth manifold is a manifold with an atlas whose transition maps are all smooth.

We have excluded infinite-dimensional (Banach) manifolds, where the invariance of domain fails and we don't have local compactness.

We shall use freely the fact that every isomorphism $S : C^\infty(Y) \to C^\infty(X)$ arises as composition with a smooth diffeomorphism $\sigma : X \to Y$; this is true even for infinite-dimensional manifolds and requires no countability assumption [7,13,4].

2. Proof of Theorem

In this Section we consider second-countable smooth manifolds only. Both Steps 1 and 2 depend on the fact that second-countable manifolds are σ-compact, while Step 3 requires that manifolds do not have too many connected components.

Given a locally compact space L we write $C_0(L)$ for the space of all real-valued continuous functions on L vanishing at infinity. If L is compact we shall omit the subscript. $C_0(L)$ is a Banach space with the sup norm

$$\|f\|_\infty = \sup_{x\in L}\bigl|f(x)\bigr|.$$

If, besides, L is a smooth manifold, we put $C_0^\infty(L) = C^\infty(L) \cap C_0(L)$. It is an easy consequence of the Stone–Weierstrass theorem that $C_0^\infty(L)$ is then dense in $C_0(L)$.

The following remark is one of the main points of the proof. The Riesz-type representation result behind it is probably well-known to specialists, but I couldn't find it in the literature. As it is usual in the real-valued setting, a linear functional $\phi : C^\infty(X) \to \mathbb{R}$ is said to be positive if it takes non-negative functions into non-negative real numbers.

Step 1. Two positive functionals on $C^\infty(X)$ which agree on $C_0^\infty(X)$ are identical.

If X is compact there is nothing to prove, so we assume X is not. Let αX denote the one-point compactification of X and consider $C^\infty(\alpha X)$ as the unitization of $C_0^\infty(X)$, so that, $C^\infty(\alpha X)$ consists of those functions $f \in C^\infty(X)$ for which $f(x)$ tends to a finite limit as $x \to \infty$.

Now, let $\phi : C^\infty(X) \to \mathbb{R}$ be a positive linear functional. It is pretty obvious that ϕ is bounded on $C^\infty(\alpha X)$ for the sup norm and since $C^\infty(\alpha X)$ is uniformly dense in $C(\alpha X)$, according to the Riesz representation there is a regular Borel measure on αX such that $\phi(f) = \int_{\alpha X} f\,d\mu$ for $f \in C^\infty(\alpha X)$. As X is the union of countably many compact subsets, there is $f \in C^\infty(X)$ such that $f(x) \to +\infty$ as $x \to \infty$ and we conclude that μ has compact support in X and, in particular, $\mu(\{\infty\}) = 0$. Hence

$$\phi(f) = \int_X f\,d\mu \quad \bigl(f \in C^\infty(\alpha X)\bigr). \tag{1}$$

Let (u_n) be an increasing sequence in $C_0^\infty(X)$ converging pointwise to 1 on X. We want to see that $\phi(u_n f) \to \phi(f)$ for every $f \in C^\infty(X)$ as $n \to \infty$. We shall prove that $\phi((1-u_n)f) \to 0$, which is enough. Look at the inner product given by

$$\langle g|f\rangle = \phi(gf) \quad \bigl(f, g \in C^\infty(X)\bigr)$$

and recall Cauchy–Schwarz inequality:

$$\bigl|\phi\bigl((1-u_n)f\bigr)\bigr| \leqslant \phi\bigl((1-u_n)^2\bigr)^{1/2}\phi\bigl(f^2\bigr)^{1/2}. \tag{2}$$

But $(1-u_n)$ and so $(1-u_n)^2$ are in $C^\infty(\alpha X)$ and applying (1) we get

$$\phi\bigl((1-u_n)^2\bigr) = \int_X (1-u_n)^2\,d\mu \to 0 \quad (n \to \infty)$$

according to Lebesgue Dominated Convergence Theorem. Thus the right-hand side of (2) goes to zero and so the left-hand one does.

In all what follows $T : C^\infty(Y) \to C^\infty(X)$ will be as in Theorem.

Step 2. There is a closed set $S \subset X$ and a homeomorphism $\tau : S \to Y$ such that $Tf(s) = f(\tau(s))$ for every $f \in C^\infty(Y)$ and $s \in S$.

This part requires a bit of Banach space theory.

It is clear that the restriction of T to $C_0^\infty(Y)$ takes values in $C_0^\infty(X)$ and preserves the sup norm:

$$\|Tf\|_\infty = \|S_f(f)\|_\infty = \|f\|_\infty.$$

Therefore, T embeds $C_0^\infty(Y)$ isometrically into $C_0^\infty(X)$ and so it extends by density to an isometric embedding of $C_0(Y)$ into $C_0(X)$ we shall denote by T_0. Let $T_0^* : C_0(X)^* \to C_0(Y)^*$ be the Banach space adjoint of T_0, so that

$$\bigl(T_0^*\phi\bigr)f = \phi(T_0 f) \quad \bigl(\phi \in C_0(X)^*,\ f \in C_0(Y)\bigr).$$

Quite clearly, T_0^* maps the (closed) unit ball of $C_0(X)^*$ onto that of $C_0(Y)^*$. As the extreme points of these balls have the form $\pm\delta_z$, with z in the corresponding space, it follows from the Hahn–Banach theorem (and the fact that T_0 maps non-negative functions into non-negative functions) that for each $y \in Y$ there is $s \in X$ such that $T_0^*\delta_s = \delta_y$. Let S be the set of those $s \in X$ such that $T_0^*\delta_s = \delta_y$ for some $y \in Y$. We define a mapping $\tau : S \to Y$ by declaring $y = \tau(s)$ if $T_0^*\delta_s = \delta_y$. We already know that τ is onto and it is easily seen that it is continuous. Let us check that τ is injective. The ensuing argument is an adaptation of [12, Proof of Theorem 2.2]. Suppose $s_1, s_2 \in S$ are such that $\tau(s_1) = \tau(s_2) = y$. Take a non-negative $f \in C_0^\infty(Y)$ such that $f(y) = 1$ and $f(z) < 1$ for $z \neq y$. Then for $i = 1, 2$, we have

$$1 = f(y) = f\bigl(\sigma(s_i)\bigr) = Tf(s_i) = f\bigl(\tau(s_i)\bigr),$$

where $\sigma : X \to Y$ is a diffeomorphism such that $Tf = f \circ \sigma$. Hence $y = \sigma(s_i)$ and so $s_1 = s_2$, as we claimed.

To prove that S is closed in X, suppose (s_n) is a sequence in S converging to $x \in X$. We have $T_0^*\delta_{s_n} \to T_0^*\delta_x$ in the weak* topology of $C_0^*(X)$ and so $T_0^*\delta_x$ is the weak* limit of the sequence (δ_{x_n}). But the weak* limit of any sequence of point evaluations must be either a point evaluation (in whose case x belongs to S) or zero and the latter cannot be for if $T_0^*\delta_x = 0$ taking any strictly positive $f \in C_0^\infty(Y)$ we would have $0 = (T_0^*\delta_x)f = \delta_x(Tf) = f(\sigma(x))$, an absurd.

Let us verify that τ is a homeomorphism. If Y (and so X and S) is compact this is obvious since $\tau : S \to Y$ is one-to-one and continuous. If Y is not compact, then neither S nor X are. As S is closed in X we can extend τ to a continuous bijection between the one point compactifications of S and Y just sending the infinity point of S to the infinity point of Y. That extension is a homeomorphism and so is τ.

We end this part by checking the formula for the value of $Tf(s)$. We already know that one has $Tf(s) = f(\tau(s))$ for $s \in S$ and $f \in C_0^\infty(Y)$ and so the functionals $f \mapsto Tf(s)$ and $f \mapsto f(\tau(s))$ are linear and positive on $C^\infty(Y)$ and agree on $C_0^\infty(Y)$. A look at Step 1 should suffice.

Step 3. $S = X$.

The key point is a standard result in algebraic topology known as the invariance of domain: every injective continuous mapping between topological manifolds without boundary of the same dimension is open (see, e.g., [6, Proposition 7.4]). To exhibit the idea, suppose that X is a connected manifold without boundary. Since S is homeomorphic with Y (and so with X itself), the invariance of domain applies to the inclusion map $S \to X$ and so S is open in X. But according to Step 1 it is also closed and so $S = X$, as required.

As we are not assuming connectedness and the invariance of domain fails for manifolds with boundary some more work must be done.

First, notice that S is a topological manifold with boundary since it is homeomorphic to Y. Also, S is a subset of X and, quite clearly, $\operatorname{Int} S \subset \operatorname{Int} X$. Let us check that one also has $\partial S \subset \partial X$. Pick $s \in \partial S$ and consider $y = \tau(s)$, which obviously lies on ∂Y. Let $f \in C^\infty(Y)$ have a unique strict global maximum at y and let us take a look at $g = Tf$. As $g = f \circ \sigma$ for certain diffeomorphism $\sigma : X \to Y$ we see that g attains a unique strict global maximum at $x = \sigma^{-1}(y)$ which lies on ∂X. But according to Step 2 one has $g(s) = f(\tau(s)) = f(y) = g(x)$ and so $s = x$ lies on the boundary of X.

It follows that $\operatorname{Int} S$ is closed in $\operatorname{Int} X$ since $\operatorname{Int} S = S \cap \operatorname{Int} X$. On the other hand, both $\operatorname{Int} S$ and $\operatorname{Int} X$ are manifolds without boundary of the same dimension and applying the invariance of domain to the inclusion $\operatorname{Int} S \to \operatorname{Int} X$ we see that $\operatorname{Int} S$ is clopen (closed and open) in $\operatorname{Int} X$. As $\operatorname{Int} X$ is dense in X we conclude that S is clopen in X.

Now, if X (hence Y and S) has finitely many connected components, then S equals X since they have the same number of components. Finally, if there are infinitely many components, by our second countability assumption, one has a topological decomposition

$$Y = \bigoplus_{n=1}^{\infty} Y_n$$

where Y_n are (the) connected, clopen subsets of Y. Put $S_n = \tau^{-1}(Y_n)$, so that $S = \bigoplus_{n=1}^\infty S_n$. Consider the function $f \in C^\infty(Y)$ which takes the value n on Y_n and set $g = Tf$. Then there is a decomposition $X = \bigoplus_{n=1}^\infty X_n$ such that $g = n$ on X_n. As $g = f \circ \tau$ on S we see that $S_n = X_n$ for every n and so $X = S$.

Step 4. $\tau : X \to Y$ is a smooth diffeomorphism.

We know that τ is a homeomorphism and smoothness follows from the fact that $f \circ \tau$ is smooth for every $f \in C^\infty(Y)$. Thus, by the inverse function theorem, it suffices to prove that $D\tau$ is an isomorphism at every point of X. Suppose $D\tau(x_0): T_{x_0}X \to T_{y_0}Y$ is not an isomorphism, where $y_0 = \tau(x_0)$. By our assumptions on the dimension the adjoint of $D\tau(x_0)$ which goes from the cotangent space of Y at y_0 to the cotangent space of X at x_0 cannot be injective and we can construct a smooth function $f: Y \to [0,1]$ having the following properties:

- The (cozero) set $V = \{y \in Y:\ f(y) > 0\}$ is diffeomorphic to $\mathbb{R}^n$ if $x_0 \in \operatorname{Int} X$ and to $\mathbb{R}^n_+$ if $x_0 \in \partial X$.
- There is a unique $y_1 \in V$ such that $f(y_1) = 1$. This is the only point inside V where Df vanishes. Moreover, $y_1 \neq y_0$.
- $Df(y_0) \circ D\tau(x_0) = 0$, but $Df(y_0) \neq 0$.

Let us take a look at $g = Tf = f \circ \tau$. Let $\sigma: X \to Y$ be a diffeomorphism such that $Tf = f \circ \sigma$. Note that if U is the cozero set of g, then $U = \tau^{-1}(V) = \sigma^{-1}(V)$ and g attains a unique strict maximum at x_1. Besides, $\sigma(x_1) = \tau(x_1) = y_1$ and x_1 is the only point of U where Dg vanishes. Of course, $x_1 \neq x_0$, since $g(x_0) < 1$. However,

$$Dg(x_0) = D(f \circ \tau)(x_0) = Df(y_0) \circ D\tau(x_0) = 0,$$

a contradiction.

This completes the proof of Theorem.

3. Remarks and examples

3.1. Measurable cardinals

While it is more or less clear that our main result should be true under much less restrictive hypotheses on the underlying manifolds, one cannot expect to prove it for arbitrarily large manifolds, as the following annotated example shows. We refer the reader to [8, Corollary 2] and [3, Theorem 3] for similar results in the bounded setting.

Example. (a) The group of automorphisms of $\mathbb{R}^I$ is reflexive if and only if I has nonmeasurable cardinal.

(b) Let I have measurable cardinal. Then, for every second-countable smooth manifold X, the group of automorphisms of $C^\infty(I \times X)$ fails to be reflexive.

Proof. Here I denotes a set of “indices” that we regard as a discrete topological space. Therefore I can be seen as a smooth manifold of dimension zero and $\mathbb{R}^I$ is just $C^\infty(I)$. Admittedly, the smooth structure in part (a) is quite trivial. In part (b) the smooth manifold $I \times X$ can be thought as a topological disjoint union of a family of copies of X, indexed by I, so that $C^\infty(I \times X) = C^\infty(X)^I$.

A **zero-one measure** on I is a countably-additive measure $\mu: 2^I \to \{0,1\}$. Trivial examples are the evaluations measures given by

$$\delta_i(A) = \begin{cases} 1 & \text{if } i \in A, \\ 0 & \text{if } i \notin A, \end{cases}$$

for some fixed $i \in I$. These are said to be **fixed**. A measure vanishing on all singletons is said to be **free**. The set I is said to have **measurable cardinal** if there is a zero-one measure on I which is free. Otherwise the cardinal of I is said to be nonmeasurable. It is a challenging problem in set-theory to know if measurable cardinals do exist or not. In any case such cardinals should be very, very large; see [9].

Positive linear functionals on $\mathbb{R}^I$ are in correspondence with finite, countably-additive measures on I. Indeed, if $\phi: \mathbb{R}^I \to \mathbb{R}$ is positive and linear, the formula $\mu(A) = \phi(1_A)$ defines a finite, countably-additive measure on 2^I. Conversely, if $\mu: 2^I \to \mathbb{R}$ is such a measure, then each $f \in \mathbb{R}^I$ is μ-integrable (in the usual sense of measure theory) and the formula

$$\phi(f) = \int_I f(i)\, d\mu(i)$$

defines a positive linear functional on $\mathbb{R}^I$. Moreover, zero-one measures on I correspond to real-valued homomorphisms on the algebra $\mathbb{R}^I$.

We now prove the “if” part of (a). Let T be a local automorphism of $\mathbb{R}^I$. For each $i \in I$ let us consider the functional $T^*\delta_i: \mathbb{R}^I \to \mathbb{R}$ defined by $(T^*\delta_i)f = \delta_i(Tf) = Tf(i)$. Clearly, $T^*\delta_i$ is a positive linear functional and it is pretty obvious that $T^*\delta_i(1_A)$ is either 0 or 1 for all $A \subset I$. Thus, if I has nonmeasurable cardinal, for each $i \in I$ there is $j \in I$ such that $\delta_j = T^*\delta_i$ and we can define a transformation τ on I just taking $\tau(i) = j$. It is then clear that $T = \tau^*$ in the sense that $Tf = f \circ \tau$ for every $f \in \mathbb{R}^I$. It remains to check that τ is a bijection. The injectivity of T implies that τ is surjective. Finally, given $j \in I$ one has $T1_j = 1_{\tau^{-1}(j)}$ and since T is a local automorphism $\tau^{-1}(j)$ must be a singleton which means that τ is injective.

Let us prove (b), which includes the “only if” part of (a) just taking X as a single point. Suppose I has measurable cardinal and let μ be a witnessing measure. We fix $\ell \in I$ and then a bijection $\tau: I \setminus \{\ell\} \to I$. We define a map $T: C^\infty(I \times X) \to \mathbb{R}^{I \times X}$ by the formula

$$Tf(i,x)=\begin{cases} f(\tau(i),x) & \text{if } i\neq \ell,\\ \int_I f(j,x)\,d\mu(j) & \text{if } i=\ell.\end{cases}$$

At this juncture it is unclear that Tf falls in $C^\infty(I\times X)$ because we don't know if $Tf(\ell,-)$ (the restriction of Tf to the ℓ-th copy of X) is smooth. Take $f\in C^\infty(I\times X)$ and put

$$I_f=\bigcap_{x\in X}\left\{i\in I:\ f(i,x)=\int_I f(k,x)\,d\mu(k)\right\}. \tag{3}$$

We claim that $\mu(I_f)=1$. Clearly, for each fixed $x\in X$ we have

$$\mu\left(\left\{i\in I:\ f(i,x)=\int_I f(k,x)\,d\mu(k)\right\}\right)=1.$$

Let D be a countable dense subset of X and

$$J_f=\bigcap_{x\in D}\left\{i\in I:\ f(i,x)=\int_I f(j,x)\,d\mu(j)\right\}.$$

As μ is countably additive we have $\mu(J_f)=1$. We claim that $I_f=J_f$. Pick $i\in J_f$, so that $f(i,x)=\int_I f(j,x)\,d\mu(j)$ for every $x\in D$. Given $y\in X$ we may take a sequence (x_n) in D converging to y. One has

$$\begin{aligned} f(i,y)&=\lim_{n\to\infty} f(i,x_n)=\lim_{n\to\infty}\int_I f(j,x_n)\,d\mu(j)\\ &=\int_I\Big(\lim_{n\to\infty} f(j,x_n)\Big)\,d\mu(j)=\int_I f(j,y)\,d\mu(j)=Tf(\ell,y),\end{aligned}$$

according to Lebesgue Dominated Convergence Theorem. In particular Tf is smooth on the ℓ-th copy of X just because f is smooth on the i-th copy.

So, certainly, T is an endomorphism of the algebra $C^\infty(I\times X)$.

Next we show that T is a local automorphism. Fix $f\in C^\infty(I\times X)$ and let I_f be as in (3). Write

$$I=\tau^{-1}(I_f)\oplus\{\ell\}\oplus\tau^{-1}\big(I_f^c\big).$$

It is easily seen that there is a bijection σ of I which agrees with τ on the set $\tau^{-1}(I_f^c)$ and sending $\tau^{-1}(I_f)\oplus\{\ell\}$ onto I_f. Thus, the map given by $Sf(i,x)=f(\sigma(i),x)$ is an automorphism of $C^\infty(I\times X)$. Moreover, $Sf=Tf$ since when $i\in\tau^{-1}(I_f^c)$ one has

$$Tf(i,x)=f\big(\tau(i),x\big)=f\big(\sigma(i),x\big)=Sf(i,x),$$

for every $x\in X$, while when $i\in\tau^{-1}(I_f)\oplus\{\ell\}$ one has

$$Tf(i,x)=Tf(\ell,x)=f\big(\sigma(i),x\big)=Sf(i,x)$$

for $x\in X$. This completes the proof of (b). □

3.2. *An open problem concerning continuous functions*

Of course one can study similar problems for continuous instead of smooth functions and, in fact, this was done in [2,5,3,12,11] and other papers; see [10, Chapter 3]. Replacing "smooth" by "continuous" everywhere in Theorem and its proof and omitting Step 4 one gets the following:

Proposition. *If X is a second-countable manifold with boundary, then the group of automorphisms of the algebra $C(X)$ is reflexive.*

A weaker result was obtained in [2, Corollary 5]. As before the result ultimately relies on the simple local structure of X through the invariance of domain. The papers [5,2,3] abound in examples showing that the preceding Proposition is not true if X is an arbitrary topological space, even if we assume compactness. The following problem is open, however.

Problem. Determine if the group of automorphisms of the Banach algebra $C(K)$ is reflexive when K is a compact metric space.

We hasten to remark that the answer is affirmative (and the proof quite simple) if one considers complex-valued instead of real-valued functions. See [12, Theorem 2.2]. The real case is probably much harder.

3.3. Note (March 5, 2012)

As I suspected, the reasoning leading to Step 1 had been used before. See [1, Appendix B].

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