1. Introduction

When an infectious disease is strongly detrimental for the population where it is spreading, such that it becomes an epidemic, then a vaccination policy should be applied to protect the susceptible individuals and control its spread. Since immunizing the whole population is impossible in most of cases, only a proportion of susceptible individuals can be immunized by vaccination. How to determine this proportion is an important problem which depends on multiple factors. A significant factor for public authorities to assess the vaccination efficiency is, in the time that the infectious disease should be allowed to survive after vaccination. In this work we provide an approach to this problem modeling epidemic spread of an infectious disease with incubation period and controlling its time to extinction by means of stochastic models, namely Sevast’yanov’s branching process.

2. The Probability Model

The Sevast’yanov’s branching process is a stochastic model which has three main features: it is a continuous-time model; the reproduction of individuals depends on their age and takes place only once at the end of the life time (see Sevast’yanov (1971)). This model is a particular case of the general branching model, which fits most adequately infectious diseases following SIR spreading scheme (see Bull and Donnelly (1995)).

Parameters of the model

- \( p(u) \): the family of contact distribution laws.
- \( u \): the proportion of immune individuals in the population, with \( 0 \leq u \leq 1 \).
- \( \{ p_{1}(w) \}_{w \geq 0} \): the family of infection distribution laws.
- \( G(\cdot) \): the distribution function of the survival time of an infected individual.

Intuitive interpretation

- \( p(u) \): the proportion of immune individuals in the population.
- \( u \): the proportion of immune individuals in the population, with \( 0 \leq u \leq 1 \).

Fixed \( u \), we have the following relation between contact distribution and infection distribution laws

\[
p_{u,w}(k) = \sum_{j=0}^{k} \binom{k}{j} u^{-j} (1-u)^{k-j} p_{j}(w).
\]

3. Time to extinction of the epidemic

Time to extinction of the epidemic means the maximal time that the infection survives into the population.

Monotonicity and continuity properties depending on proportion \( u \)

- The greater is the proportion of the immune individuals, more probable the disease disappears faster.
- Minor changes in the proportion of the immune individuals generates minor changes in time to extinction.
- The quantile of order \( p, 0 \leq p \leq 1 \), of time to extinction has the same monotonicity and continuity properties (see left graphic in Figure 2).

4. Determining vaccination policies

We suppose that before vaccination, every healthy individual which is in contact with an infected individual is non-immune. At an arbitrary time \( t \), the vaccination process of susceptible individuals starts and finishes at time \( t_{f} \) (see left graphic in Figure 1). Therefore \( t_{1} \leq t_{f} \) is the time that it is taken for immunization, called the vaccination period. After this period, every vaccinated individual is immune and we suppose that this proportion is \( \alpha \).

Vaccination based on the quantiles of the time to extinction

For fixed \( p \) and \( t_{f} \), with \( 0 < p < 1 \) and \( t_{f} > 0 \), we look for \( \alpha \), which guaranties that the infectious disease becomes extinct, with probability greater than or equal to \( p \), not later than time \( t_{f} \) after the vaccination process ended. We define the optimal vaccination policy by

\[
\alpha_{opt} = \alpha(p, r, z) = \inf \{ \alpha: \alpha_{opt}(\alpha) \geq p \},
\]

where \( z \) is the greatest integer number smaller or equal to the expected number of infected individuals at time \( t_{f} \) providing the vaccination policy has not been applied and \( \alpha_{opt}(\cdot) \) is the distribution function of the time to extinction of a SBP started at time \( 0 \) with \( z \) individuals. In order to determine \( \alpha_{opt} \), we approximate \( u_{opt} \) by means of a simulation-based method (see Martinez and Slavtchova-Bojkova (2005)).

Analyzing the control measures for avian influenza in Vietnam

In 2006 an outbreak of avian influenza in domestic birds starts widespread itself in the southern part of Vietnam and becomes extinct in 14th January 2007 (see OIE (2007) and the right graphic of Figure 1). In order to apply the simulation-based method, we consider that \( G(\cdot) \) is the d.f. of a gamma distribution with mean 5 and shape 16, to guarantee that the survival period in 90% of individuals is between 3 and 7 days. For each \( u \) satisfying \( \{ p(u) \}_{u \geq 0} \) follows a Poisson distribution with parameter \( \lambda \), being \( \lambda > 0 \). Since the number of infected individuals at the first outbreak is 80 and after the incubation period the total number of infected individuals is 413, we can estimate the rate \( m_{u} \) for \( u \). Also we consider the vaccination period between 15th and 19th December. Moreover, we approximate the number of individuals incubating the virus at this date by \( 2112 \times 0.41/141/500 \). Therefore, if we consider \( p = 0.91 \) and \( t_{f} = 30 \), we obtain that \( \alpha_{opt}(0.91, 30, 2132) = 0.97 \).

References


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Figure 1. Left: A possible evolution of infected individuals. Right: Numbers of infected domestic birds in Vietnam.

Figure 2. Left: Empirical d.f. of time to extinction for some \( u \). Right: Histogram of simulated extinction time for \( u = 0.97 \).