

In the second part of the article *Comme Appelé du Néant – As If Summoned from the Void: The life of Alexandre Grothendieck* by Allyn Jackson, published in the Notices of the AMS, November 2004, there is a small misprint in the reference of a partial translation of *Récoltes et Semailles* in Spanish at <http://kolmogorov.unex.es/~navarro/res/> because the tilde  $\sim$  lacks in such reference. I would be very grateful if you warn it to your readers.

This web page contains an ongoing translation of some interesting (in my opinion) parts of *Récoltes et Semailles* and *La Clef des Songes*. My interest in Grothendieck arose in the seventies, when studying at the University of Salamanca, where the undergraduate studies in Mathematics had a highly coherent structure devised by Prof. Sancho Guimerá. He taught two courses on Algebraic Geometry, and I attended them being 20 and 21 years old. Two days a week, from 12:00 to no fixed ending hour (but usually at three or four o'clock in the afternoon, the last hours in the corridors or the streets), he showed us how the simple and natural ideas of Grothendieck pervade mathematics. How separable extensions of a field  $k$  may be understood as coverings of a one point space  $\text{Spec } k$ , the localization process being the change of the base field and the globalization process encoding Galois theory. How sheaf cohomology is well-suited for both Algebraic Geometry and Algebraic Topology. How any morphism  $X \rightarrow T$  may be understood as a family of spaces parameterized by  $T$ , so that absolute statements about a space  $X$  really refer to the projection  $X \rightarrow p$  onto the one point space. How absolute statements always have relative versions about families  $X \rightarrow T$ , which greatly simplify the theory because of the freedom they provide. How any morphism  $T \rightarrow X$  may be understood as a point of  $X$  (parameterized by  $T$ ) and that obviously such functor of points of  $X$  fully determines the whole structure of  $X$ , so reducing definitions of morphisms to the silly case of sets – provided that they are natural definitions, i.e. functorial – and constructions to the problem of representing a functor. How representability theorems are very simple in the case of categories of abelian sheaves, and they directly provide the existence of dualizing sheaves and dualizing complexes in the case of smooth curves (representing  $H^1(X, \mathcal{M})^*$ , Riemann-Roch's theorem), smooth varieties (Serre's duality), topological manifolds (Poincaré's duality) and proper morphisms (Grothendieck's duality). How toposes may unify Algebraic Geometry, Arithmetic and Topology, so that the scent of these ideas one day should pervade Mathematics. And over all, how deeply rooted we have the wish of canonical, natural and simple definitions, statements and proofs, and that such yearning may be always overwhelmingly accomplished. These few years with Prof. Sancho Guimerá have fed my mathematical life since.

In 1985 I was studying homotopy theory and a friend gave me a photocopy of Grothendieck's *À la Poursuite des Champs*. When reading it, I saw that a few questions were quite close to my own work and I kept up a brief correspondence with Grothendieck on them. Then he sent me a copy of *Récoltes et Semailles* and something echoed inside me when I read that he presents the relationship of any man with the spiritual goods under a double aspect. On the one hand,

the luminous aspect (the passion for knowledge) represented by the figure of a child. On the other hand, the obscure aspect of *la peur et de ses antidotes vaniteux, et les insidieux blocages de la créativité qui en dérivent*, ReS A3 (the fear and vanity, and the insidious blockade of creativity stemmed from them). I feel that a deep and valuable lesson of the main body of *Récoltes et Semailles* is summarized in the last paragraph but one of the Introduction (II.10 xxii):

*Si quelque chose pourtant est saccagé et mutilé, et desamorcé de sa force originelle, c'est en ceux qui oublient la force qui repose en eux-mêmes et qui s'imaginent saccager une chose à leur merci, alors qu'ils se coupent seulement de la vertu créatrice de ce qui est à leur disposition comme elle est à la disposition de tous, mais nullement à leur merci ni au pouvoir de personne.* (If something is despoiled and mutilated, defused of its initial force, it is in those unaware of the force resting inside them, who think they are despoiling something at their mercy, while they are cutting themselves off from the creative virtue of that which is at their disposal as this virtue is at the disposal of everyone, but not at their mercy nor under the power of anyone).

But I was mainly moved by the Preface, *Promenade à travers une oeuvre – ou l'enfant et la Mère* (Promenade through an oeuvre – or the child and the Mother), written after the echoes of the first distribution of *Récoltes et Semailles*. First by the innocence of the child making mathematics, of our children repeatedly asking *why?* and *what is?* instead of the weary and clumsy *what's this for?* of adults. And above all by the silently, quite, attentive, feminine attitude of listening our whispering interior voice and readiness to accept it, remembering me the disposition: “be it into me according to your word”. I realized that (following the path of Descartes, Pascal, Leibnitz,...) Grothendieck has contributed to embed Mathematics as a significant part of a broader spiritual adventure: man's self-conscience unfolding. Hence I strongly recommend this *Promenade* to my students of Algebraic Geometry. They may listen the author of the theories they are studying, speaking about their deep roots and sense, a unique and wonderful luck. Since most of my students don't understand French, I began to translate it into Spanish.

When I wrote to Grothendieck that I missed the exigence of solitude in any creative work, explained in the *Promenade*, since the friendly atmosphere at the University of Salamanca was essential for me, he said me in a letter (June of 1987):

*Vous soulignez, à juste titre, la difficulté psychique de la création solitaire... C'est là la situation qui a été la mienne plus ou moins pendant toute ma vie, depuis mon enfance, tant sur le plan de la création mathématique, que dans mon itinéraire spirituel... Et sans cela, rien de grand ne s'accomplit, ni dans l'aventure individuelle, ni dans l'aventure collective – que ce soit au plan intellectuel, ou au plan spirituel... Au niveau spirituel, la plus grande oeuvre (à*

*mes yeux) qu'un homme ait accomplie, était la Passion du Christ et sa mort sur la croix... Cette oeuvre était et ne pouvait être que solitaire. Et même c'était la solitude suprême, car Dieu Lui-même s'est retiré, pour que l'Oeuvre s'accomplisse sans le secours d'une consolation.* (You remark the psychic difficulty of any lonely creation... Such has been my situation all my life, since my childhood, in the mathematical creation and in my spiritual itinerary... And without that, nothing great is accomplished, in the individual adventure, nor in the collective adventure – at the intellectual level or at the spiritual level... At the spiritual level, the biggest oeuvre (in my eyes) accomplished by a man, was the Passion of the Christ and his death on the cross. This oeuvre was and should be solitary. It was even the supreme solitude, since God moved away, so that the Oeuvre is accomplished without the help of any consolation.)

I understood that actually any great creative labour has a hard period of solitude, and I was touched by his tough life. A few months ago, I got *La Clef des Songes* and I was deeply moved by his embedding of mathematics into man's religious adventure. He says in page 100: *Les lois mathématiques peuvent être découvertes par l'homme, mais elles ne sont créées ni par l'homme ni même par Dieu. Que deux plus deux égale quatre n'est pas un décret de Dieu, qu'Il aurait été libre de changer en deux plus deux égale trois, ou cinq. Je sens les lois mathématiques comme faisant partie de la nature même de Dieu – une partie infime, certes, la plus superficielle en quelque sorte, et la seule qui soit accessible à la seule raison.* (Mathematical laws are discovered by man, but they are not created by man, not even by God. Two plus two equals four is not a God's decree, that He may change in two plus two equals three, or five. I feel mathematical laws as a part of the nature of God – certainly a negligible part, the most superficial in a certain sense, and the only accessible to the reason by itself.)

Since 1990, I haven't heard from him, except for some comments included in Allyn Jackson's article *Comme Appelé du Néant*. Finally, let me say a few words about the disarmingly simple question *what is a metre?*. It sounds infantile, since the metre is a mere convention taught at school. But we should not underestimate a question raised by a man who has given so tremendous answers to the question *what is a space?*, which also seemed to be well known. The metre is an arbitrary length unit and, given the reluctance of Grothendieck to any artificial concept, it is plausible to assume that he is asking for the possibility of *natural* units. And this is not a ridiculous question. The existence of natural units is implicitly assumed, for example, when we say that “the speed of light is equal for any observer”, a statement grounding the Theory of Relativity. Since such speed depends on the time and length units, this statement is full-sense only if the existence of *universal* units is assumed, adopted now here and at the ends of the universe at any time. This is a strong hypothesis that, combined with other “common sense” geometric assumptions, is capable of giving account of the semi-riemannian structure of relativity.

I am aware that this letter is longer than a usual rectification letter. Please, let me know any paragraph you consider not interesting for your readers, freely correct my poor English and ask me any clarification or explanation you wish.

Sincerely yours

Juan A. Navarro González

navarro@unex.es