

Formulaire des Six opérations de Grothendieck et leurs compatibilités

$$\begin{array}{c}
 X \\
 \downarrow f \\
 Y
 \end{array}
 \left.
 \begin{array}{l}
 \begin{array}{l}
 \mathbb{F} \\
 \otimes \\
 \mathcal{F}
 \end{array} \\
 \begin{array}{l}
 \text{RHom} \\
 \text{RHom}(\mathcal{F}, -) \\
 \text{Rf}^* \quad \text{Lf}^* \\
 \text{Rf}_! \quad \text{Rf}_!
 \end{array}
 \end{array}
 \right\}
 \begin{array}{l}
 \text{RHom}, \Gamma_X \\
 \tau_X h_{\mathcal{F}}
 \end{array}$$

$$\begin{array}{l}
 1 \otimes \mathcal{F} \circ \mathcal{F} \circ 1 \circ \mathbb{F} \xrightarrow{\tau_X \circ id} \text{RHom}(\mathbb{1}, \mathcal{F}) \simeq \mathcal{F} \quad h_1 \simeq id \\
 \mathcal{F} \circ \mathcal{F} \simeq \mathcal{F} \circ \mathcal{G} \quad \tau_{\mathcal{F} \circ \mathcal{G}} \simeq \tau_{\mathcal{G}} \circ \tau_{\mathcal{F}} \\
 (\mathcal{F} \circ \mathcal{G}) \circ \mathcal{H} \simeq \mathcal{F} \circ (\mathcal{G} \circ \mathcal{H}) \quad \tau_{\mathcal{F} \circ \mathcal{G}} \simeq \tau_{\mathcal{G}} \circ \tau_{\mathcal{F}} \\
 \text{RHom}(\mathcal{F} \circ \mathcal{G}, \mathcal{H}) \simeq \text{RHom}(\mathcal{F}, \text{RHom}(\mathcal{G}, \mathcal{H}))
 \end{array}$$

$$\text{RHom}(\mathcal{F}, \mathcal{G} \circ \mathcal{H}) \leftarrow \text{RHom}(\mathcal{F}, \mathcal{G}) \otimes \mathcal{H}$$

$$\text{RHom}(\mathcal{F}, \mathcal{G}) \simeq \text{R}\Gamma_X(\text{RHom}(\mathcal{F}, \mathcal{G}))$$

$$\text{Hom}(\mathcal{F}, \mathcal{G}) = \text{H}^0 \text{RHom}(\mathcal{F}, \mathcal{G}) = \text{H}^0 \text{R}\Gamma_X(\text{RHom}(\mathcal{F}, \mathcal{G}))$$

$$\begin{array}{l}
 \text{Lf}^*(\mathcal{F} \circ \mathcal{G}) = \text{Lf}^* \circ \mathcal{L}\mathcal{F} \\
 \text{Lf}^* \circ \mathcal{L}\mathcal{F} = \tau_{\mathcal{L}\mathcal{F}} \circ \mathcal{L}\mathcal{F}^* \quad \text{R}\Gamma_X(\mathcal{F}) = \text{RHom}(\mathbb{1}, \mathcal{F}) \\
 \text{R}\Gamma_X \simeq \text{Rf}_! \circ \text{Rf}_*
 \end{array}$$

$$\begin{array}{c}
 X \quad \mathcal{F} \\
 \downarrow f \\
 Y \quad \mathcal{G}
 \end{array}
 \quad \text{Rf}_!(\text{RHom}_X(\mathcal{F}, \mathcal{G})) \simeq \text{RHom}_Y(\mathcal{G}, \text{Rf}_!(\mathcal{F}))$$

$$\boxed{ \text{Rf}_! \text{RHom}_X(\mathcal{F}, \text{Rf}_!^! \mathcal{G}) \simeq \text{RHom}_Y(\mathcal{R}^0 \mathcal{F}, \mathcal{G}) }$$

$$\boxed{ \text{Hom}_X(\mathcal{F}, \text{Rf}_!^! \mathcal{G}) \simeq \text{Hom}_Y(\mathcal{R}^0 \mathcal{F}, \mathcal{G}) }$$

$$\boxed{ \text{Rf}_!(\mathcal{F} \circ \mathcal{L}\mathcal{F}^!(\mathcal{G})) \simeq \mathcal{R}^0(\mathcal{F}) \otimes \mathcal{G} }$$

$$\text{RHom}(\mathcal{F} \circ \mathcal{L}\mathcal{F}^!(\mathcal{G}), \mathcal{H}) \simeq \tau_{\mathcal{L}\mathcal{F}^!(\mathcal{G})} \circ \text{RHom}(\mathcal{F}, \mathcal{H})$$

$$\text{Lf}^*(\mathcal{F} \circ \mathcal{L}\mathcal{F}^!(\mathcal{G})) \simeq \text{Lf}^*(\mathcal{F}) \circ \mathcal{L}\mathcal{F}^!(\mathcal{G})$$

$$\text{RHom}_X(\mathcal{L}\mathcal{F}^!(\mathcal{G}), \text{Rf}_!^! \mathcal{H}) \simeq \text{RHom}_Y(\mathcal{R}^0 \mathcal{F}, \mathcal{H})$$

$$\text{Rf}_!(\mathcal{F} \circ \mathcal{L}\mathcal{F}^!(\mathcal{G})) \leftarrow \text{Rf}_!(\mathcal{F}) \otimes \mathcal{G}$$

$$\begin{array}{ccc}
 F & X & \xleftarrow{h} & Y' \\
 \downarrow f & & & \downarrow f' \\
 Y & & \xleftarrow{g} & Y'
 \end{array}$$

$$\begin{array}{l}
 Lg^* Rf_!(F) \simeq Rf'_! Lh^*(F) \\
 Rf_! Rg_* \simeq Rh_* Rf'_!
 \end{array}$$

$$Lg^* Rf_!(F) \iff Rf'_* Lh^*(F)$$

$$Lf^*(F) \xleftarrow{\Omega_{X/Y}} Lf'^*(F) \otimes L\Omega_{X/Y}$$

$$\begin{array}{ccc}
 F & X & \xrightarrow{f'} & Y \\
 \downarrow f & & & \downarrow f' \\
 S & & \xrightarrow{g} & Y'
 \end{array}$$

$$F \otimes S \simeq Lg^*(F) \otimes Lf'^*(S)$$

$$Rh_*(F \otimes S) \simeq Rf_*(F) \otimes Rg_*(S)$$

Kawada

is  $i: g$  lisse  
 is  $i: f$  lisse  
 $f: X \rightarrow Y, X, Y$  lisse  
 $\downarrow$   
 $S, Y'$  lisse

Notes de Alexandre Grothendieck d'un mini cours donné à Mormoiron dans le Vaucluse (France) en mai 1983 donnant un formulaire des six opérations cohomologiques et de leurs compatibilités.