DEO Group
Differential Equations and Optimization

Vladimir V. Goncharov

CIMA, Departamento de Matemática, Universidade de Évora

First Joint Meeting Evora-Extremadura on Mathematics
Badajos, Spain
January, 14, 2016

Group consists of

- 12 effective members
- 9 collaborators
- 3 PhD students

4 research lines

- Boundary Value Problems
- Calculus of Variations and Optimal Control
- Numerical Methods
- Variational, Multialued and Functional Analysis

Cooperation

- International Cooperation
  - University of Castilla-La Mancha (Spain)
  - University of Padua (Italy)
  - University of Napoles (Italy)
  - Carnegi Mellon University (USA)
  - Dorodnicym Computing Centre of RAS (Moscow, Russia)
  - Institute of Systems Dynamics and Control Theory of RAS (Irkutsk, Russia)
- National Cooperation
  - CMAF
  - Universidade Nova de Lisboa
  - Universidade de Aveiro
  - Universidade de Porto
  - Universidade de Minho
- Local cooperation
  - Centro de Geofísica de Évora
  - Instituto de Ciências Agrárias Mediterráneas, UE

Boundary Value Problems

Team:

- Effective members
  - F. Minhos (coordinator)
  - A.I. Santos
- Collaborator
  - J. Fialho
- PhD students
  - H. Carrasco
  - R. Carrapinho

Boundary Value Problems. Objectives and achievements

Fields of research

- Boundary Value Problems for:
  - high order ordinary differential equations;
  - integro-differential equations;
  - fractional differential equations;
  - partial differential equations;
  - discrete and time scale high order equations

  including problems with nonlocal and functional boundary data
- Applications in:
  - mechanics;
  - engineering;
  - medicine

Boundary Value Problems. Objectives and achievements

Main recent achievements

- Existence and location results are obtained for a fourth order functional boundary value problem;
- Existence, location and multiplicity results for higher order boundary value problems are obtained in continuous, discrete and in time scale cases;
- Nonlocal and functional higher order boundary value problems are studied;
- Nonlinear impulsive fourth order boundary value problems are studied;
- Comparison technique and arguments based on the Degree Theory are developed

Boundary Value Problems. Example of Application

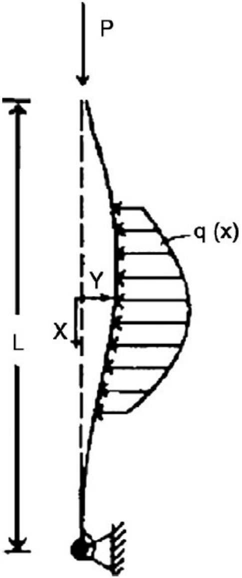
Beam-column deformations

BV problems for high order ordinary differential equations permit to study the mechanical deformations of the human spine, which is under various loading conditions such as aircraft ejections or vehicle crush situations. One can take in consideration also some initial deformations due, for instance, certain form of scoliosis. The displacement $y_1(x)$ of the beam-column is modeled with a fourth order nonlinear equation

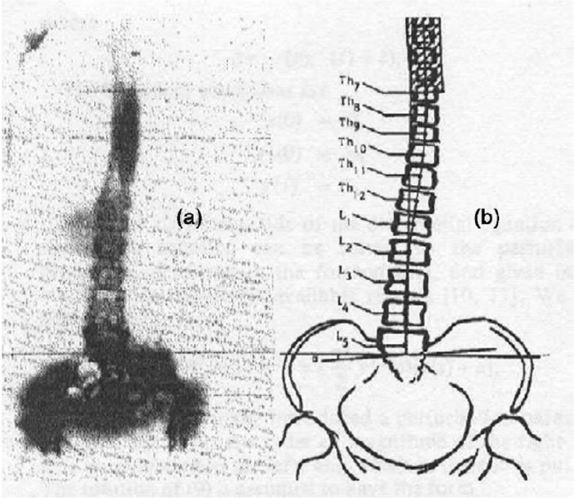
$$EIy_1^{(4)} + Py_1'' = q(y_1''') - Py_0'',$$

where EI is the flexural rigidity of the beam-column, P is the axial load, $y_0(x)$ is the initial displacement and $q(\cdot)$ is some continuous function of the third derivative of the displacement meaning the transverse forces. The boundary condition here includes the restriction on the maximal curvature (the second derivative) of the column, i.e., we have a functional BV problem

Boundary Value Problems. Example of Application



Boundary Value Problems. Example of Application



Boundary Value Problems. Some recent publications

- **J. Fialho** and **F. Minhós**, High order periodic impulsive problems, *Dyn. Syst. Dif. Equat. Appl. AIMS Proc.* (2015), 446-454
- **F. Minhós** and **J. Fialho**, Existence and multiplicity of solutions in fourth order BVPs with unbounded nonlinearities, *Discr. Cont. Dynam. Syst.*, Supplement (2013), 555-564
- **J. Fialho** and **F. Minhós**, On higher order fully periodic boundary value problems, *J. Math. Anal. Appl.* 395 (2012), 616-625
- M. Grossinho, **F. Minhós** and **A. Santos**, A note on a class of problems for a higher order fully nonlinear equation under one sided Nagumo type condition, *Nonlinear Anal.* 70 (2009), 4027-4038
- **F. Minhós**, On some third order nonlinear boundary value problems: existence, location and multiplicity results, *J. Math. Anal. Appl.* 339/2 (2008), 1342-1353

Calculus of Variations and Optimal Control

Team:

- Effective members
  - A. Ornelas (coordinator)
  - C. Carlota
  - L. Bandeira
  - L. Bicho
- Collaborators
  - M Vornicescu
  - S. Chá

Calculus of Variations and Optimal Control. Objectives and achievements

Fields of research

- Existence theorems for solutions of scalar as well as vector variational and Optimal Control problems;
- Developing of Direct Methods;
- Regularity of minimizers;
- Necessary conditions of optimality

Calculus of Variations and Optimal Control. Objectives and achievements

Main recent achievements

- Existence of scalar minimizers for some highly discontinuous nonconvex nonautonomous multiple integrals was proved;
- Validity of DuBois-Reymond differential inclusion under pointwise constraints on state and gradient was established;
- Some necessary conditions of minimum in variational and optimal control problems (in both convex and cnonconvex cases) under pointwise state and velocity constraints are obtained;
- A pointwise constrained version of the A.A. Lyapunov's convexity theorem for single integrals was proved;
- The relations between rank-one-convexity and quasiconvexity of symmetric 2×2-matrices, quasiconvexity of fourth-degree polynomials and laminates for symmetric 2×2-gradients were studied

Calculus of Variations and Optimal Control. Some recent publications

- **C. Carlota, S. Chá** and **A.Ornelas**, A pointwise constrained version of the Liapunov convexity theorem for single integrals, *NoDEA* 20 (2013), 273-293
- **L. Bandeira** and **A. Ornelas**, On the characterization of laminates for 2x2 symmetric gradients, *J. Convex Anal.* 18 (2011), 37-58
- **L. Bicho** and **A. Ornelas**, Existence of minimizers for nonautonomous highly discontinuous scalar multiple integrals with pointwise constrained gradients, *Discr Cont Dyn Sys.*, A 29 (2011), 439-451

Calculus of Variations and Optimal Control. Some recent publications

- **C. Carlota** and **A. Ornelas**, The DuBois-Reymond differential inclusion for autonomous optimal control problems with pointwise-constrained derivatives, *Discr Cont Dyn Sys.*, A 29 (2011), 467-484
- **L. Bicho** and **A. Ornelas**, Radially increasing minimizing surfaces or deformations under pointwise constraints on positions and gradients, *Nonlinear Anal.* 74 (2011), 7061-7070

Numerical Methods

Team:

- Effective members
  - V. Bushenkov (coordinator)
  - F. Carapau
- Collaborators
  - A. Lotov (Russia)
  - P. Correia
  - M Pires
- PhD student
  - A. Jorge

Numerical Methods. Objectives and achievements

Fields of research

- Numerical methods in finite as well as infinite dimensional optimization;
- Multiobjective optimization;
- Computer realization of the numerical schemes;
- Mathematical modeling of real physical, chemical etc. processes;
- Applications to fluid mechanics and to various regional problems

Numerical Methods. Objectives and achievements

Main recent acchievements

- Numerical algorithms for approximating Pareto frontier for some integer multiobjective problems are constructed;
- An algorithm for determination of space-time slip distribution for seismic sources is given;
- Mathematical models of fluid mechanics are analysed, developed and applied to hemodynamics and to other medicine problems;
- The developed methods are applied to forest management, to water allocation and to other problems of the Alentejo region

Example 1 (modeling of the liquid flows interacted with the surrounding matter)

Numerical analysis of the blood flow through the human tissues such as the filtration blood through the kidneys. The results can be applied in medicine, e.g., in the kidneys dialysis

The bood flow in such a situation can be described by the following hydrodynamical equations

Numerical Methods. Examples of Application

Example 1 (modeling of the liquid flows interacted with the surrounding matter)

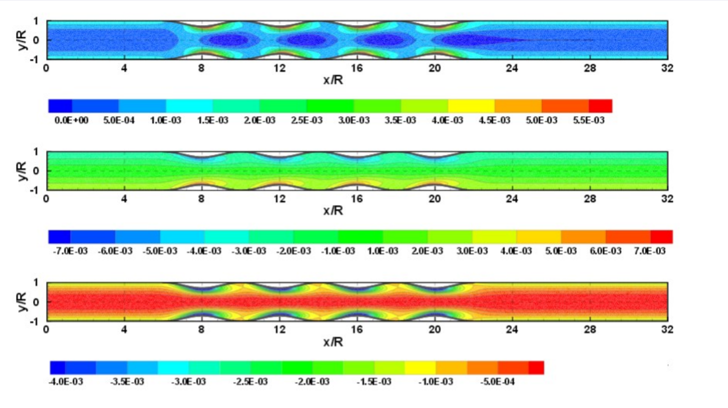
Classical Johnson-Segalmann Model Summary

$$
\begin{aligned}
\operatorname{div} \boldsymbol{u} &= 0 \\
\rho \frac{d\boldsymbol{u}}{dt} &= -\boldsymbol{\nabla} p + \operatorname{div} \mathbf{T} \\
\mathbf{T} &= \mathbf{T}_s + \mathbf{T}_e \\
\mathbf{T}_s &= 2\mu_s \mathbf{D} \\
\frac{\partial \mathbf{T}_e}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\, \mathbf{T}_e &= \frac{2\mu_e}{\lambda}\mathbf{D} - \frac{1}{\lambda}\mathbf{T}_e + (\mathbf{W}\mathbf{T}_e - \mathbf{T}_e\mathbf{W}) + a(\mathbf{D}\mathbf{T}_e + \mathbf{T}_e\mathbf{D})
\end{aligned}
$$

$$a \in [-1; 1]$$

Numerical Methods. Examples of Applications

Example 1 (modeling of the liquid flows interacted with the surrounding matter)

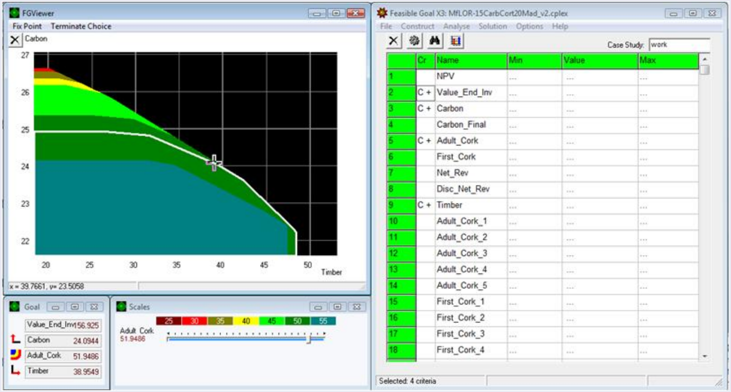


Example 2 (modeling of the forests management)

Based on a multicriterion optimization method a system of the decision suport for the forests management was created in collaboration with Instituto Superior de Agronoimia in Lisbon

This method can be afterwards successively applied in other situations, for instance, for modeling of the management of the hidro-resourses (such as Alqueva)

Example 2 (modeling of the forest management)



Numerical Methods. Some recent publications

- J. Borges, J. Garcia-Gonzalo, **V. Bushenkov**, M. McDill, S. Marques and M. Oliveira, Addressing multi-criteria forest management with Pareto Frontier methods: an application in Portugal, *Forest Science* 60 (2014), 63-72
- T. Bodnar, **M. Pires** and J. Janela, Blood flow simulation using traceless variant of Johnson-Segalman viscoelastic model, *Math. Model. of Nat. Phen.* 9 (2014), 117 141
- **F. Carapau** and J. Janela, A one-dimensional model for unsteady axisymmetric swirling motion of a viscous fluid in a variable radius straight circular tube, *Int. J. Eng. Science* 72 (2013), 107-116
- R. Fragoso, **V. Bushenkov** and C. Marques, Multi-objective water allocation in the Alqueva Region, *New Medit.* 9 (2010), 28-35
- **F. Carapau**, One-dimensional viscoelastic fluid model where viscosity and normal stress coeffients depend on the shear rate, *Nonlinear Anal.: Real World Appl.* 11 (2010) 4342-4354

Variational, Multivalued and Functional Analysis

Team:

- Effective members
  - V. Goncharov (coordinator)
  - G. Carita
  - F. Pereira
  - T. Santos
- Collaborators
  - N. Freire
  - E. Zappale (Italy)
  - V. Roshchina (Australy)

Fields of research

- Application of the methods of Nonlinear Functional Analysis in CV
  - Existence theorems for vector variational problems;
  - Relaxation and other qualitative properties of solutions;
  - Homogenization;
  - Strong Maximum Principle
- Convex and Nonsmooth Analysis
  - Geometry of convex solids in Hilbert (Banach) spaces;
  - Weakly convex sets;
  - Properties of the generalized subgradients;
  - Properties of conic problems;
  - Metric regularity;
  - Best approximation problems
- Time Optimal Control in Hilbert and Banach spaces
- Multivalued Analysis and Differential Inclusions
- Abstract Functional Analysis. Interpolation of linear operators

Variational, Multivalued and Functional Analysis

Main recent achievements

- Some theorems on relaxation for vector problems of Calculus of Variations including a nonlinear optimal design problem for various growth assumptions are proved;
- 3D-2D dimensional reduction for micromagnetics thin films is proposed;
- Existence of a solution in some vector variational problem with the knitting boundary data is proved, and the application to a plastic surgery problem is given;
- New versions of the Strong Maximum Principle for elliptic variational functional with a generalized symmetry assumptions are proposed;
- Quantitative characteristics of convex bodies in a Hilbert space are introduced and studied;
- Pointwise version of weak convexity in Hilbert spaces is introduced and unified with strong convexity;

Main recent achievements

- Notion of directed subdifferential (in the Demyanov's sense) is extended to a class of functions more general than quasidifferentiable;
- Finite precision analysis of the 17^{th} Smale problem is given;
- Complementarity partition result for ill-posed multifold conic systems is obtained;
- Condition measure of smoothing algorithm for matrix games is calculated by using the metric regularity;
- Bodies with zero resistance and invisibility in geometrical optics are studied;

Variational, Multivalued and Functional Analysis

Main recent achievements

- Geometric conditions for well-posedness of a best approximation problem in a Hilbert space are given;
- Regularity of the value function in a kind of Optimal Time Control problem near the target is studied;
- Relations between well-posedness of the optimal time control problem (with the constant convex dynamics) and the Fréchet differentiability of the value function in Banach spaces are established;
- Existence of an exposed solution to a differential inclusion under a Hölder assumption is proved in two-dimensional space;
- Stochastic approach to Differential Inclusions is developed;
- Some compactness properties in general topological spaces are studied;
- Some results on Banach lattices and interpolation of linear operators are obtained

Variational, Multivalued and Functional Analysis. Example of Application

Example (plastic surgery)

We study a vector variational problem with non traditional boundary conditions, which appear in the plastic surgery, namely, in modeling of the woman's breast

We are interested in knitting of the breast tissues. Considering the breast as a superelastic body we should minimize the energy functional, which depends on the gradient of the tissue displacement $u(\cdot)$:

$$\int_{\Omega} W(\nabla u(x))\, dx$$

where W is a polyconvex integrand satisfying natural growth assumptions, including coercivity

Variational, Multivalued and Functional Analysis. Example of Application

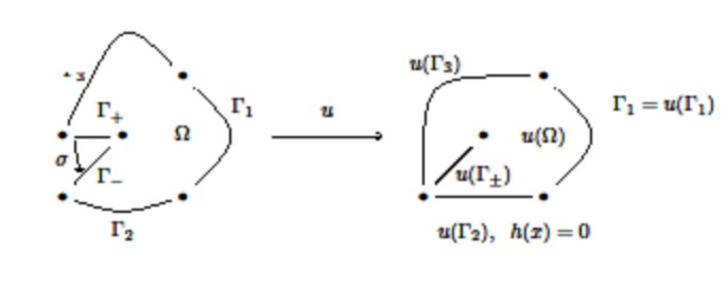
Example (plastic surgery)

Furthermore, we should guarantee that the various parts of the breast surface were knitted carefully. Namely,

- one of the piece Γ_1 coincides with a part of the woman's chest, which was not cut. So, the displacement here should verify $u(x) = x$;
- another piece Γ_2 should be knitted with a fixed chest surface, so the boundary condition on Γ_2 is $h(u(x)) = 0$;
- one more piece Γ_3 is devided into two parts Γ^+ and Γ^-, which should be knitted one to other, and the respective condition is $u(\sigma(x)) = u(x)$ being $\sigma(\cdot)$ an unknown function;
- finally, one part of the breast surface Γ_4 is left free allowing a flexibility of the tissue during the knitting process

Variational, Multivalued and Functional Analysis. Example of Application

Example (plastic surgery)



Variational, Multivalued and Functional Analysis. Some recent publications

- G. Colombo and **V. Goncharov**, Brownian motion and exposed solutions of differential inclusions, *NoDEA* 20 (2013), 323-343
- A. Plakhov and **V. Roshchina**, Fractal bodies invisible in 2 and 3 direction, *Discr. Cont. Dynam. Syst* A 33 (2013), 1615-1631
- **N. Freire**, Characterizatios of distributios in a open subset of R^n. Positive distributions, *Global J. Pure and Appl. Math.* 9 (2013), 505-512
- **V. Goncharov** and **F. Pereira**, Geometric conditions for regularity in a time-minimum problem with constant dynamics, *J. Conv. Anal.* 19 (2012), 631-669

Variational, Multivalued and Functional Analysis. Some recent publications

- **G. Carita** and **E. Zappale**, 3D-2D dimensional reduction for a nonlinear optimal design problem with perimeter penalization, *Comptes Rendus Math. Acad. Sci. Paris* 350 (2012), 1011-1016
- J. Peña and **V. Roshchina**, A complementarity partition theorem for multifold conic systems, *Math. Progr* 142 (2012), 579-589
- **G. Carita**, I. Fonseca and G. Leoni, Relaxation in $SBV_p(\Omega; S^{d-1})$, *Calc. Var. Partial Dif. Equat.* 42 (2011), 211-255
- **V. Goncharov** and **T. Santos**, Local estimates for minimizers of some convex integral functional of the gradient and the Strong Maximum Principle, *Set-Valued Var. Anal.* 19 (2011), 79-202

PhD Theses

Last five years 6 persons concluded and successfully defended their PhD Theses, and 4 of them became assistant professors of the Mathemaical Department and the effective members of CIMA, while 2 are collaborators because they work in other institututions

PhD Theses

List of PhD Theses

- **S. Chá** (2013) Convex and non-convex problems in Calculus of Variations. Advisors: A. Ornelas and C. Carlota
- **L.B. Bicho** (2012) Existence and regularity for minimizers of convex and nonconvex multiple integrals of the calculus of variations under pointwise-constraints. Advisor: A.Ornelas
- **T. Santos** (2012) Some versions of the Strong Maximum Principle for elliptic integral functional. Advisors: V. Goncharov and A. Cellina
- **J. Fialho** (2012) Existence, localization and multiplicity results for nonlinear and functional higher order BVP. Advisor: F. Minhos
- **F. Pereira** (2010) Geometric conditions of regularity in some kind of minimal time problems. Advisor: V. Goncharov
- **L. Bandeira** (2009) Analysis of new situations for quasiconvexity versus rank-one convexity in 2x2 and other dimensions. Advisors: P. Pedregal and A. Ornelas