Hereditarily Normaloid Operators

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Abstract

A Banach space operator $T \in B(\mathcal{X})$ is said to be hereditarily normaloid, $T \in \mathcal{H}N$, if every part of T is normaloid; $T \in \mathcal{H}N$ is totally hereditarily normaloid, $T \in \mathcal{T}HN$, if every invertible part of T is also normaloid; and $T \in \mathcal{C}HN$ if either $T \in \mathcal{T}HN$ or $T - \lambda I$ is in $\mathcal{H}N$ for every complex number λ . Class $\mathcal{C}HN$ is large; it contains a number of the commonly considered classes of operators. We study operators $T \in \mathcal{C}HN$, and prove that the Riesz projection associated with a $\lambda \in \text{iso}\sigma(T)$, $T \in \mathcal{C}HN \cap B(\mathcal{H})$ for some Hilbert space \mathcal{H} , is self-adjoint if and only if $(T - \lambda I)^{-1}(0) \subseteq (T^* - \overline{\lambda}I)^{-1}(0)$. Operators $T \in \mathcal{C}HN$ have the important property that both T and the conjugate operator T^* have the single-valued extension property at points λ which are not in the Weyl spectrum of T; we exploit this property to prove a-Browder and a-Weyl theorems for operators $T \in \mathcal{C}HN$.

^{*} To Professor Carl Pearcy on his seventieth birthday.

Key words: Banach space, Weyl's theorem, single valued extension property, hereditarily normaloid operators, paranormal and *-paranormal operators.