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Metric Ellipses in Minkowski Planes

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Abstract

An ellipse in $\mathbb{R}^2$ can be defined as the locus of points for which the sum of the Euclidean distances from the two foci is constant. In this paper we will look at the sets that are obtained by considering in the above definition distances induced by arbitrary norms.

Let $X = (\mathbb{R}^2, \|\,\|)$. For $x \in X$, $\|x\| = 1$, and $c > 2$ next figure shows the metric ellipse $E(x, c) = \{y \in X : \|x + y\| + \|x - y\| = c\}$ for different norms:

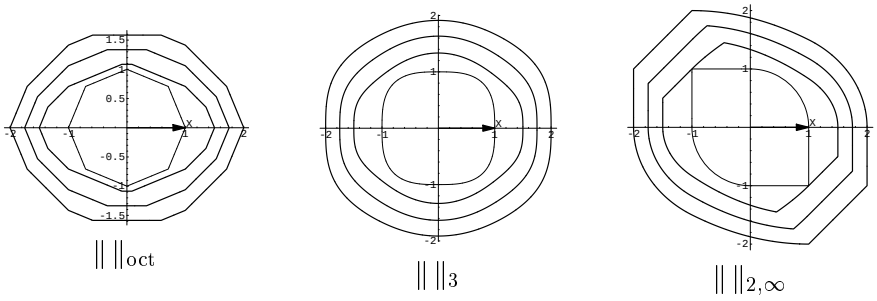

Figure 1: Metric ellipses $E((1,0),c)$ for $c = 3$, 3.5, 4 and several norms.

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References

[1] Alonso, J., Ullán, A., Moduli in normed linear spaces and characterization of inner product spaces, *Arch. Math.,* **59** (1992), 487–495.

[2] Baronti, M., Casini, E., Papini, P.L., Triangles inscribed in a semicircle, in Minkowski planes, and in normed spaces, *J. Math. Anal. Appl.,* **252** (2000), 124–146.

[3] Lindenstrauss, J., On the modulus of smoothness and divergent series in Banach spaces, *Michigan Math. J.,* **10** (1963), 241–252.

[4] Martini, H., Swanepoel, K.J., Weiss, G., The geometry of Minkowski spaces - A survey. Part I, *Expo. Math.,* **19** (2) (2001), 97–142.

[5] Martini, H., Swanepoel, K.J., Weiß, G., The Fermat-Torricelli problem in normed planes and spaces, *J. Optim. Theory Appl.,* **115** (2) (2002), 283–314.

[6] Ullán, A., "Modulos de Convexidad y Lisura en Espacios Normados", Ph.D. Thesis, Matemáticas-Universidad de Extremadura, Badajoz (Spain), 1991