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A New Proof of James' Sup Theorem

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Abstract

We provide a new proof of James' sup theorem for (non necessarily separable) Banach spaces. One of the ingredients is the following generalization of a theorem of Hagler and Johnson : *"If a normed space E does not contain any asymptotically isometric copy of $\ell 1$, then every bounded sequence of E' has a normalized $\ell 1$-block sequence pointwise converging to 0"*.

Keywords: James' sup theorem, asymptotically isometric copy of $\ell 1$, Hagler and Johnson's theorem, block sequences, reflexive Banach spaces, Axiom of Choice.

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