

Non-trivial Derivations on Commutative Regular Algebras

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1. ABSTRACT

Necessary and sufficient conditions are given for a (complete) commutative algebra that is regular in the sense of von Neumann to have a non-zero derivation. In particular, it is shown that there exist non-zero derivations on the algebra $L(M)$ of all measurable operators affiliated with a commutative von Neumann algebra M , whose Boolean algebra of projections is not atomic. Such derivations are not continuous with respect to measure convergence. In the classical setting of the algebra $S[0, 1]$ of all Lebesgue measurable functions on $[0, 1]$, our results imply that the first (Hochschild) cohomology group $H^1(S[0, 1], S[0, 1])$ is non-trivial.

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