

## On the Moore-Penrose Inverse in $C^*$ -algebras

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### ABSTRACT

In this article, two results regarding the Moore-Penrose inverse in the frame of  $C^*$ -algebras are considered. In first place, a characterization of the so-called reverse order law is given, which provides a solution of a problem posed by M. Mbekhta. On the other hand, Moore-Penrose hermitian elements, that is  $C^*$ -algebra elements which coincide with their Moore-Penrose inverse, are introduced and studied. In fact, these elements will be fully characterized both in the Hilbert space and in the  $C^*$ -algebra setting. Furthermore, it will be proved that an element is normal and Moore-Penrose hermitian if and only if it is a hermitian partial isometry.

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