EXTRACTA MATHEMATICAE Vol. **21**, Núm. 2, 167 – 190 (2006)

Homogeneous Polynomial Vector Fields of Degree 2 on the 2–Dimensional Sphere

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(Presented by Manuel de León)

AMS *Subject Class.* (2000): 34C35, 58F09, 34D30 *Received July* 17, 2006

1. Abstract

Let X be a homogeneous polynomial vector field of degree 2 on $\mathbb{S}^2$ having finitely many invariant circles. Then, we prove that each invariant circle is a great circle of $\mathbb{S}^2$, and at most there are two invariant circles. We characterize the global phase portrait of these vector fields. Moreover, we show that if X has at least an invariant circle then it does not have limit cycles.

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Key words: invariant circle, invariant great circle, polynomial vector field.

*The authors are partially supported by a MCYT grant MTM2005–06098–C02–01 and by a CIRIT grant number 2005SGR00550.

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