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A Note on the Range of Generalized Derivation

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1. Abstract

Let $\mathcal{L}(H)$ denote the algebra of bounded linear operators on a complex separable and infinite dimensional Hilbert space H. For $A, B \in \mathcal{L}(H)$, the generalized derivation $\delta_{A,B}$ associated with (A, B), is defined by $\delta_{A,B}(X) = AX - XB$ for $X \in \mathcal{L}(H)$. In this note we give some sufficient conditions for A and B under which the intersection between the closure of the range of $\delta_{A,B}$ respect to the given topology and the kernel of δ_{A^*,B^*} vanishes.

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