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Linear Maps Preserving the Generalized Spectrum *

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Abstract: Let H be an infinite-dimensional separable complex Hilbert space and $\mathcal{B}(H)$ the algebra of all bounded linear operators on H. For an operator T in $\mathcal{B}(H)$, let $\sigma_g(T)$ denote the generalized spectrum of T. In this paper, we prove that if $\phi : \mathcal{B}(H) \to \mathcal{B}(H)$ is a surjective linear map, then ϕ preserves the generalized spectrum (i.e. $\sigma_g(\phi(T)) = \sigma_g(T)$ for every $T \in \mathcal{B}(H)$) if and only if there is $A \in \mathcal{B}(H)$ invertible such that either $\phi(T) = ATA^{-1}$ for every $T \in \mathcal{B}(H)$, or $\phi(T) = AT^{tr}A^{-1}$ for every $T \in \mathcal{B}(H)$. Also, we prove that $\gamma(\phi(T)) = \gamma(T)$ for every $T \in \mathcal{B}(H)$ if and only if there is $U \in \mathcal{B}(H)$ unitary such that either $\phi(T) = UTU^*$ for every $T \in \mathcal{B}(H)$ or $\phi(T) = UT^{tr}U^*$ for every $T \in \mathcal{B}(H)$. Here $\gamma(T)$ is the reduced minimum modulus of T.

Key words: reduced minimum modulus, generalized spectrum, Jordan isomorphism,linear preserver problems.

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