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Characterization of Bessel Sequences

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Abstract: Let $\mathcal{H}$ be a separable Hilbert space, $L(\mathcal{H})$ be the algebra of all bounded linear operators of $\mathcal{H}$ and $Bess(\mathcal{H})$ be the set of all Bessel sequences of $\mathcal{H}$. Fixed an orthonormal basis $E = \{e_k\}_{k\in\mathbb{N}}$ of $\mathcal{H}$, a bijection $\alpha_E : Bess(\mathcal{H}) \longrightarrow L(\mathcal{H})$ can be defined. The aim of this paper is to characterize $\alpha_E^{-1}(\mathcal{A})$ for different classes of operators $\mathcal{A} \subseteq L(\mathcal{H})$. In particular, we characterize the Bessel sequences associated to injective operators, compact operators and Schatten p-classes.

Key words: Bessel sequences, frames, bounded linear operators.

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