

Isometric Embeddings and Universal Spaces

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Abstract: We show that if a separable Banach space Z contains isometric copies of every strictly convex separable Banach space, then Z actually contains an isometric copy of every separable Banach space. We prove that if Y is any separable Banach space of dimension at least 2, then the collection of separable Banach spaces which contain an isometric copy of Y is analytic non Borel.

Key words: isometrically universal space, strictly convex norm, well-founded tree.

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REFERENCES

- [1] ARGYROS, S., DODOS, P., Genericity and amalgamation of classes of Banach spaces, *preprint*.
- [2] BOSSARD, B., Coanalytic families of norms on a separable Banach space, *Illinois J. Math.* **40** (2) (1996), 162–181.
- [3] BOSSARD, B., A coding of separable Banach spaces. Analytic and coanalytic families of Banach spaces, *Fund. Math.* **172** (2002), 117–152.
- [4] BOSSARD, B., GODEFROY, G., KAUFMAN, R., Hurewicz’s theorems and renormings of Banach spaces, *J. Funct. Anal.* **140** (1) (1996), 142–150.
- [5] DODOS, P., FERENCZI, V., Some strongly bounded classes of Banach spaces, *preprint*.
- [6] GODEFROY, G., Universal spaces for strictly convex Banach spaces, *Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat.* **100** (1-2) (2006), 137–146.
- [7] GODEFROY, G., KALTON, N.J., Lipschitz-free Banach spaces, *Studia Math.* **159** (1) (2003), 121–141.
- [8] GODEFROY, G., KAUFMAN, R., YAHDI, M., The topological complexity of a natural class of norms on Banach spaces, *Ann. Pure Appl. Logic* **111** (2001), 3–13.
- [9] KECHRIS, A., “Classical Descriptive Set Theory”, Springer-Verlag, New York, 1995.

- [10] LINDENSTRAUSS, J., Notes on Klee's paper: "Polyhedral sections of convex bodies", *Israel J. Math.* **4** (1966), 235–242.
- [11] ODELL, E., SCHLUMPRECHT, T., A separable reflexive space universal for the uniformly convex Banach spaces, *Math. Ann.* **335** (4) (2006), 901–916.
- [12] ROLEWICZ, S., "Metric Linear Spaces", Monografie Mat. 56, PWN-Polish Scientific Publishers, Warsaw, 1972.
- [13] ROSENTHAL, H.P., On applications of the boundedness principle to Banach space theory according to J. Bourgain, Séminaire d'Initiation à l'Analyse 18e, Paris 6 (1979), 5.01–5.14.