On Operators that Preserve the Radon-Nikodým Property [*]

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Abstract: We consider a certain class $\mathcal{RN}_+$ of operators that preserve the Radon-Nikodým property. Conjugate operators in $\mathcal{RN}_+$ can be characterized as those operators T such that the kernel $N(T^*+K^*)$ has the Radon-Nikodým property for every compact operator K. A construction by J. Bourgain involving infinite convolution products of measures in the Cantor group provides examples of operators $T\colon L_1 \to L_1$ in the class $\mathcal{RN}_+$. As an application, we show the existence of Banach spaces which are $\mathcal{L}_1$-spaces, have the Radon-Nikodým property and contain infinite-dimensional reflexive subspaces.

Key words: Banach space, Radon-Nikodým property, Asplund space, $\mathcal{L}_1$-space, compact perturbation.

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