

Topological Open Problems in the Geometry of Banach Spaces *

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Abstract: We survey a brief account of topological open problems inside the area of renormings of Banach spaces. All of them are related with the Stone's theorem on the paracompactness of metric spaces since it is our scalpel to find out the rigidity condition for the renorming process. All of them have been collected in our recent monograph [21] which is the main source for the present survey article. We also present recent results showing the connection between Stone's theorem and Deville's master lemma for locally uniformly rotund renormings.

Key words: locally uniformly rotund norm, Kadec norm, Fréchet norm, network, measure of non compactness.

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