Uniqueness of Invariant Hahn-Banach Extensions

Pradipta Bandyopadhyay, Ashoke K. Roy

Stat–Math Division, Indian Statistical Institute, 202, *B. T. Road, Kolkata* 700108, *India,*
pradipta@isical.ac.in, royashoke@hotmail.com

Abstract: Let ℓ be a linear functional on a subspace Y of a real linear space X provided with a sublinear functional p with $\ell \leq p$ on Y. If $\mathcal{G}$ is an abelian semigroup of linear transformations $T : X \to X$ such that $T(Y) \subseteq Y$, $p(Tx) \leq p(x)$ and $\ell(Ty) = \ell(y)$ for all $T \in \mathcal{G}$, $x \in X$ and $y \in Y$ respectively, then a generalization of the classical Hahn-Banach theorem asserts that there exists an extension $\widetilde{\ell}$ of ℓ, $\widetilde{\ell} \leq p$ on X and $\widetilde{\ell}$ remains invariant under $\mathcal{G}$. The present paper investigates various equivalent conditions for the uniqueness of such extensions and these are related to nested sequences of p-balls, a concept that has proved useful in recent years in dealing with such extensions. The results are illustrated by a variety of examples and applications.

Key words: Sublinear functionals, nested sequences of (p-) balls, invariant Hahn-Banach extensions.

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