

Continuous Multilinear Operators on $C(K)$ Spaces and Polymeasures [†]

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Abstract: Every continuous k -linear operator from a product $C(K_1) \times \cdots \times C(K_k)$ into a Banach space X (K_i being compact Hausdorff spaces) admits a Riesz type integral representation

$$T(f_1, \dots, f_k) := \int (f_1, \dots, f_k) d\gamma,$$

where γ is the representing polymeasure of T , i.e., a set function defined on the product of the Borel σ -algebras $\text{Bo}(K_i)$ with values in X^{**} which is separately finitely additive. As in the linear case, the interplay between T and its representing polymeasure plays an important role. The aim of this paper is to survey some features of this relationship.

Key words: Multilinear operators, spaces of continuous functions, tensor products of Banach spaces, polymeasures.

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