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On the Existence of Prolongations of Connections by Bundle Functors

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Abstract: We construct canonically a general connection $A^F(\Gamma,\nabla)$ on $Fp:FY\to FM$ from a general connection Γ on a fibred manifold $p:Y\to M$ by means of a projectable classical linear connection ∇ on Y, where $F:\mathcal{M}f\to\mathcal{VB}$ is a vector bundle functor. In the case of a not necessarily vector bundle functor $F:\mathcal{M}f\to\mathcal{FM}$ we find some simple equivalent condition on the existence of a general connection $A(\Gamma,\nabla)$ on $Fp:FY\to FM$ from a general connection Γ on $Y\to M$ by means of a projectable classical linear connection ∇ on Y. We present a construction of a classical linear connection $A^F(\nabla)$ on FY from a projectable classical linear connection ∇ on Y for any fiber product preserving bundle functor $F:\mathcal{FM}_m\to\mathcal{FM}$. We characterize bundle functors $F:\mathcal{FM}_{m,n}\to\mathcal{FM}$ which admit a construction of a classical linear connection $A(\nabla)$ on FY from a projectable classical linear connection ∇ on Y. We characterize gauge bundle functors $F:\mathcal{VB}_{m,n}\to\mathcal{FM}$ which admit a construction of a classical linear connection $A(D,\nabla)$ on FE from a linear general connection D on $E\to M$ by means of a classical linear connection ∇ on M.

Key words: General connection, classical linear connection, (vector) (gauge) bundle functor, fiber product preserving bundle functor, Weil algebra, natural isomorphism, natural (gauge) operator.

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