Trivial Units in Commutative Group Algebras

PETER DANCHEV

13 *General Kutuzov Street, bl.* 7, *floor* 2, *flat* 4, 4003 *Plovdiv, Bulgaria*
pvdanchev@yahoo.com

Abstract: Let G be an arbitrary abelian group and let R be a commutative unitary ring of arbitrary characteristic. A necessary and sufficient condition is given for when all units in the group ring RG are trivial provided that either $\text{supp}(G) \cap \text{inv}(R) \neq \emptyset$ or RG is modular. In particular, we establish a comprehensive characterization by finding a criterion when RG has only trivial units provided that $\text{char}(R)$ is a positive number greater than 1. These achievements strengthen results due to Karpilovsky (Arch. Math. Basel, 1983), Herman-Li-Parmenter (Can. Math. Bull., 2005) and the author (Math. Commun., 2005).

Key words: normed units, trivial units, group rings, indecomposable rings, reduced rings, idempotents, nilpotents.

AMS *Subject Class.* (2000): 16U60, 16S34, 20K21.

REFERENCES

[1] D.B. COLEMAN, Idempotents in group rings, *Proc. Amer. Math. Soc.* **17** (4) (1966), 962.

[2] P.V. DANCHEV, Warfield invariants in abelian group rings, *Extracta Math.* **20** (3) (2005), 233 – 241.

[3] P.V. DANCHEV, On the trivial units in finite commutative group rings, *Math. Commun.* **10** (2) (2005), 143 – 147.

[4] P.V. DANCHEV, A note on trivial units in abelian group rings, *An. Univ. Bucureşti Mat.* **54** (2) (2005), 229 – 234.

[5] A. HERMAN, Y. LI, Trivial units for group rings over rings of algebraic integers, *Proc. Amer. Math. Soc.* **134** (3) (2006), 631 – 635.

[6] A. HERMAN, Y. LI, M.M. PARMENTER, Trivial units for group rings with G-adapted coefficient rings, *Canad. Math. Bull.* **48** (1) (2005), 80 – 89.

[7] G. HIGMAN, The units of group rings, *Proc. Lond. Math. Soc.* **46** (1940), 231 – 248.

[8] G. KARPILOVSKY, On units in commutative group rings, *Arch. Math. (Basel)* **38** (1982), 420 – 422.

[9] G. KARPILOVSKY, On finite generation of unit groups of commutative group rings, *Arch. Math. (Basel)* **40** (1983), 503 – 508.

[10] G. KARPILOVSKY, "Unit Groups of Group Rings", Longman Scientific & Technical, Harlow, 1989.

[11] W.L. MAY, Group algebras over finitely generated rings, *J. Algebra* **39** (1976), 483 – 511.

[12] T.ZH. MOLLOV, N.A. NACHEV, Unit groups of commutative group rings, *Compt. Rend. Acad. Bulg. Sci.* **57** (5) (2004), 9 – 12.

[13] D.S. PASSMAN, "The Algebraic Structure of Group Rings", Wiley – Interscience, New York-London-Sydney, 1977.

[14] J. RITTER, S.K. SEHGAL, Trivial units in *RG*, *Math. Proc. R. Ir. Acad.* **105** (1) (2005), 25 – 39.

[15] S.K. SEHGAL, "Topics in Group Rings", Marcel Dekker, Inc., New York, 1978.

[16] W.D. ULLERY, On isomorphism of group algebras of torsion abelian groups, *Rocky Mountain J. Math.* **22** (3) (1992), 1111 – 1122.