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Perturbations of Operators Satisfying a Local Growth Condition

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Abstract: A Banach space operator $T \in B(\mathcal{X})$ satisfies a local growth condition of order m for some positive integer m , $T \in \mathrm{loc}(G_m)$, if for every closed subset F of the set of complex numbers and every x in the glocal spectral subspace $\mathcal{X}_T(F)$ there exists an analytic function $f : \mathbb{C} \setminus F \to \mathcal{X}$ such that $(T-\lambda)f(\lambda) \equiv x$ and $||f(\lambda)|| \leq K[\mathrm{dist}(\lambda, F)]^{-m}||x||$ for some $K > 0$ (independent of F and x). Browder-Weyl type theorems are proved for perturbations by an algebraic operator of operators which are either $\mathrm{loc}(G_m)$ or polynomially $\mathrm{loc}(G_m)$.

Key words: Banach space, local growth condition, single valued extension property, Browder-Weyl theorems, Riesz operator, perturbation, polynomially locally (G_m) operator.

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