# Invariant Metric $f$-Structures on Specific Homogeneous Reductive Spaces

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*Abstract*: For homogeneous reductive spaces $G/H$ with reductive complements decomposable into an orthogonal sum $\mathfrak{m} = \mathfrak{m}_1 \oplus \mathfrak{m}_2 \oplus \mathfrak{m}_3$ of three $Ad(H)$-invariant irreducible mutually inequivalent submodules we establish simple conditions under which an invariant metric $f$-structure $(f, g)$ belongs to the classes $\mathbf{G_1f}$, $\mathbf{NKf}$, and $\mathbf{Kill\,f}$ of generalized Hermitian geometry. The statements obtained are then illustrated with four examples. Namely we consider invariant metric $f$-structures on the manifolds of oriented flags $SO(n)/SO(2) \times SO(n-3)$ ($n \geq 4$), the Stiefel manifold $SO(4)/SO(2)$, the complex flag manifold $SU(3)/T_{max}$, and the quaternionic flag manifold $Sp(3)/SU(2) \times SU(2) \times SU(2)$.

*Key words*: Homogeneous reductive space, $f$-structure, invariant structure, nearly Kähler structure, flag manifold.

AMS *Subject Class.* (2000): 53C15, 53C30, 53C10.

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