Representations of Codimension One Non-Abelian Nilradical Lie Algebras

MAHMOUD RAWASHDEH, G. THOMPSON

Department of Mathematics, University of Findlay, Findlay, OH 45840, USA rawshdeh@findlay.edu

Department of Mathematics, The University of Toledo, 2801 W. Bancroft St., Toledo, OH 43606, USA, thompson@math.utoledo.edu

Presented by Manuel de León

Received January 31, 2008

Abstract: A Theorem is proved that shows that for a solvable Lie algebra \mathfrak{h} of dimension n+2 whose nilradical is codimension one and for which the nilradical has a one-dimensional derived algebra there is a subgroup of $\operatorname{GL}(n+2,\mathbb{R})$ whose Lie algebra is isomorphic to \mathfrak{h} . The Theorem helps to give a more conceptual understanding of the classification of the algebras in dimensions four, five and six. Finally the main Theorem is applied to a particularly interesting class of algebras for which the nilradical is isomorphic to the five-dimensional Heisenberg algebra.

 $Key\ words\colon$ Low-dimensional Lie algebras, right-invariant vector field, Lie algebra extension, codimension one nilradical.

AMS Subject Class. (2000): 17B30, 17B56, 22E15, 22E60.

References

- R. GHANAM, I. STRUGAR, G. THOMPSON, Matrix representations for low dimensional Lie algebras, *Extracta Math.* 20 (2) (2005), 151–184.
- [2] R. GHANAM, G. THOMPSON, S. TONON, Representations for six-dimensional nilpotent Lie algebras, *Hadronic J.* 29 (3) (2006), 299–317.
- [3] S. HELGASON, "Differential Geometry, Lie Groups and Symmetric Spaces", Academic Press, New York-London, 1978.
- [4] N. JACOBSON, "Lie Algebras", Interscience Publishers, New York-London, 1962.
- [5] V.V. MOROZOV, Classification of nilpotent Lie algebras in dimension six, Izv. Vyssh. Uchebn. Zaved., Mat. 4 (5) (1958), 161–171.
- [6] G.M. MUBARAKZYANOV, The classification of the real structure of fivedimensional Lie algebras, *Izv. Vyssh. Uchebn. Zaved.*, *Mat.* 3 (34) (1963), 99-106.
- [7] G. MUBARAKZYANOV, Classification of solvable Lie algebras iof sixth order with a non-nilpotent basis element, *Izv. Vyssh. Uchebn. Zaved.*, *Mat.* 4 (35) (1963), 104–116.

181

- [8] J. PATERA, R.T. SHARP, P. WINTERNITZ, H. ZASSENHAUS, Invariants of real low dimension Lie algebras, J. Math. Phys. 17 (1976), 986–994.
- [9] M. RAWASHDEH, G. THOMPSON, The inverse problem for six-dimensional codimension two nilradical Lie Algebras, J. Math. Phys. 47 (11) (2006), 112901, 29 pp.
- [10] P. TURKOWSKI, Low-dimensional real Lie algebras, J. Math. Phys. 29 (10) (1988), 2139-2144.
- [11] P. TURKOWSKI, Solvable Lie algebras of dimension six, J. Math. Phys. 31 (6) (1990), 1344–1350.