Limits and the Ext Functor

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Abstract: We show the identity $\mathrm{Ext}(\lim_{\leftarrow} X_\alpha, \mathbb{R}) = \lim_{\rightarrow} \mathrm{Ext}(X_\alpha, \mathbb{R})$ for projective limits of quasi-Banach spaces X_α. The proof is derived from a pull-back lemma asserting that a topologically exact sequence $0 \to \mathbb{R} \to E \to Z \to 0$ of locally pseudoconvex spaces is the pull-back of an exact sequence of quasi-Banach spaces. Among the consequences we show that exact sequences $0 \to \mathbb{R} \to E \to Z \to 0$ of locally pseudoconvex spaces come induced by quasi-linear maps, which extends a result of Kalton for Fréchet spaces; and that projective limits of K-spaces are K-spaces.

Key words: Twisted sums, Ext functor, K-space.

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