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Type II-Λ-Weak Radon-Nikodym Property in a Banach Space Associated with a Compact Metrizable Abelian Group

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Abstract: Let G be a compact metrizable abelian group with normalized Haar measure λ, Γ the dual group of G and Λ a subset of Γ. Let X be a Banach space and $f : G \longrightarrow X$ be a Pettis integrable function with respect to λ. It has been shown that the set $\{\hat{f}(\gamma) : \gamma \in \Lambda\}$ of the Fourier coefficients of f is a relatively norm compact subset of X. We have shown by a counter-example that the converse of this result is not true, in general. We have introduced the idea of type II-Λ-Weak Radon-Nikodym property (type II-Λ-WRNP) of X and have shown that the converse is true for X having this property when Λ is a Riesz set. We have also obtained several necessary and sufficient conditions for X to possess this property when Λ is a Riesz set.

Key words: Compact metrizable abelian group, Pettis integrable functions, Riesz sets, type II-Λ-weak Radon-Nikodym property.

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