

Topics in Linear Isometries of Function Algebras[†]

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Abstract: After a brief presentation of Banach-Stone's and Kahane-Zelazko's results on weighted composition operators, the attention is concentrated on the iteration of surjective and non-surjective linear isometries acting on complex Banach algebras. Appealing to results by Grothendieck, Bourgain and Lotz, the set of iterates is replaced by a strongly continuous semigroup of linear isometries.

Key words: Character, composition operator, Gelfand spectrum, strongly continuous semigroup.

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REFERENCES

- [1] R. ARENS, I.M. SINGER, Function values as boundary integrals, *Proc. Amer. Math. Soc.* **5** (1954), 735–745.
- [2] S. BANACH, “Théorie des Opérations Linéaires”, Monografie Matematyczne, PWN, Warsaw, 1932; Dover, New York, 1955.
- [3] H. BART, S. GOLDBERG, Characterizations of almost periodic strongly continuous groups and semigroups, *Math. Ann.* **236** (1978), 105–116.
- [4] A. BERNARD, Algèbres quotients d'algèbres uniformes, *C.R. Acad. Sci. Paris, Sér. A* **272** (1971), 1101–1104.
- [5] J. BOURGAIN, Propriétés de relèvement et projections dans les espaces L^1/H_0^1 et H^∞ , *C.R. Acad. Sci. Paris, Sér. A-Math.* **291** (11) (1980), 607–609.
- [6] R.B. BURCKEL, “Characterizations of $C(X)$ among its Subalgebras”, Lecture Notes in Pure and Applied Mathematics, 6, Marcel Dekker, Inc., New York, 1972.
- [7] R.B. BURCKEL, Iterating analytic self-maps of discs, *Amer. Math. Monthly* **88** (1981), 396–407.
- [8] K. DE LEEUW, W. RUDIN, J. WERMER, The isometries of some function spaces, *Proc. Amer. Math. Soc.* **11** (1960), 694–698.

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- [9] S. EILENBERG, Banach space methods in topology, *Ann. of Math.* **43** (1942), 568–579.
- [10] P. FATOU, Séries trigonométriques et séries de Taylor, *Acta Math.* **30** (1906), 335–400.
- [11] R.J. FLEMING, J.E. JAMISON, “Isometries on Banach Spaces: Function Spaces”, Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics, 129, Chapman and Hall/CRC, Boca Raton, 2003.
- [12] T. FRANZONI, E. VESENTINI, “Holomorphic Maps and Invariant Distances”, *Notas de Matemática (Mathematical Notes)*, 69, North Holland Publishing Co., Amsterdam-New York, 1980.
- [13] J.B. GARNETT, “Bounded Analytic Functions”, *Pure and Applied Mathematics*, 96, Academic Press, Inc., New York-London, 1981.
- [14] A. GLEASON, A characterization of maximal ideals, *J. Analyse Math.* **19** (1967), 171–172.
- [15] L.A. HARRIS, Banach algebras with involution and Möbius transformations, *J. Funct. Anal.* **11** (1972), 1–16.
- [16] L.A. HARRIS, Bounded symmetric homogeneous domains in infinite dimensional spaces, in “Proceedings in Infinite Dimensional Holomorphy”, T.L. Hayden and T.J. Suffridge (eds), University of Kentucky, 1973, *Lecture Notes in Mathematics*, 364, Springer, Berlin, 1974, 13–40.
- [17] K. HOFFMAN, “Banach Spaces of Analytic Functions”, Prentice-Hall, Englewood Cliffs, N.J., 1962.
- [18] K. HOFFMAN, Analytic functions and logmodular Banach algebras, *Acta Math.* **108** (1962), 271–317.
- [19] W. HOLSZTYŃSKI, Continuous mappings induced by isometries of spaces of continuous functions, *Studia Math.* **26** (1966), 133–136.
- [20] J.-P. KAHANE, W. ZELAZKO, A characterization of maximal ideals in commutative Banach algebras, *Studia Math.* **29** (1968), 339–343.
- [21] H.P. LOTZ, Uniform convergence of operators on L^∞ and similar spaces, *Math. Z.* **190** (1985), 207–220.
- [22] H.P. LOTZ, Semigroups on L^∞ and H^∞ , in “One-Parameter Semigroups of Positive Operators”, R. Nagel (ed.), *Lecture Notes in Math.*, 1184, Springer-Verlag, Berlin-Heidelberg-New York-Tokyo, 1986, 54–59.
- [23] D.E. MARSHALL, Blaschke products generate H^∞ , *Bull. Amer. Math. Soc.* **82** (1976), 494–496.
- [24] E. MICHAEL, Locally multiplicatively-convex topological algebras, *Mem. Amer. Math. Soc.* **11** (1952).
- [25] J. MILNOR, “Dynamics in One Complex Variable”, *Annals of Mathematics Studies*, 160, Princeton University Press, Princeton, N.J., 2006.
- [26] N. NAGASAWA, Isomorphisms between commutative Banach algebras with an application to rings of analytic functions, *Kodai Math. Sem. Rep.* **11** (1959), 162–188.
- [27] R.R. PHELPS, Extreme points in function algebras, *Duke Math. J.* **32** (1965), 267–278.

- [28] W. RUDIN, Boundary values of continuous analytic functions, *Proc. Amer. Math. Soc.* **7** (1956), 808–811.
- [29] W. RUDIN, “Real and Complex Analysis”, McGraw Hill, New York, 1966.
- [30] W. RUDIN, “Functional Analysis”, McGraw Hill, New York, 1973.
- [31] B. RUSSO, H. DYE, A note on unitary operators in C^* -algebras, *Duke Math. J.* **33** (1966), 413–416.
- [32] S. SAKAI, “ C^* -Algebras and W^* -Algebras”, Springer-Verlag, New York-Heidelberg-Berlin, 1971.
- [33] M.H. STONE, Applications of the theory of Boolean algebras to general topology, *Trans. Amer. Math. Soc.* **41** (1937), 375–481.
- [34] E.L. STOUT, “The Theory of Uniform Algebras”, Bogden and Quigley, Inc., Tarrytown-on-Hudson, N.Y., 1971.
- [35] E. VESENTINI, On the Banach-Stone theorem, *Adv. Math.* **112** (1995), 135–146.
- [36] E. VESENTINI, Periodic points and non-wandering points of continuous dynamical systems, *Adv. Math.* **134** (1998), 308–327.
- [37] E. VESENTINI, Weighted composition operators and the Gleason-Kahane-Zelazko theorem, *Adv. Math.* **191** (2005), 423–445.
- [38] E. VESENTINI, The Gleason-Kahane-Zelazko theorem and function algebras, *Rend. Lincei Mat. Appl.* (9), **16** (2005), 87–108; *Errata*, ibd, (9) **16** (2005), 211.
- [39] E. VESENTINI, Weighted composition operators and locally convex algebras, *Science in China, Ser. A, Mathematics* **48** (2005), 32–46.
- [40] E. VESENTINI, Periodic and almost periodic linear isometries of Hardy spaces, *Rend. Acc. Naz. Sci. XL Mem. Mat. Appl.* (5) **29** (1) (2005), 455–472.
- [41] E. VESENTINI, Linear isometries of some function algebras, *Rend. Lincei Mat. Appl.* **18** (2007), 235–255.
- [42] E. VESENTINI, Characters and idempotents acting on Banach algebras, *Tuebingen Berichte zur Funktionalanalysis* **16** (2007/2008), 451–473.