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Subnormality and Moment Problems†

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Abstract: There exists a strong connection between the concept of (sub)normality and that of moment problem. They interact very often, sometimes in a subtle, unexpected way. It is possible to use a (sub)normality result, providing eventually a spectral measure, used to solve a moment problem. Conversely, there are situations when the solution to a moment problem leads to the existence of a normal extension for some operators. The present work endeavor to present several results sustaining the interplay mentioned above, as well as the necessary background to understand those phenomena, both is a bounded or a unbounded context.

Key words: Bounded and unbounded (sub)normal operators; spectral measure; moment problems; algebraic sets; algebras of fractions.

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