Torsion Graph over Multiplication Modules

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Abstract: For a commutative ring R, the torsion graph of an R-module M is $\Gamma(M)$ whose vertices are nonzero torsion elements of M, and two distinct vertices x and y are adjacent if and only if $[x : M][y : M]M = 0$. In this article we show that if $S = R \setminus Z(M)$, then $\Gamma(M)$ and $\Gamma(S^{-1}M)$ are isomorphic for a multiplication R-module M. Also we prove that for a multiplication R-module M, if $\Gamma(M)$ is uniquely complemented, then $S^{-1}M$ is von Neumann regular or $\Gamma(M)$ is a star graph.

Key words: Torsion graph, multiplication module, von Neumann regular module.

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