EXTRACTA MATHEMATICAE Vol. **24**, Núm. 3, 251 – 258 (2009)

Linear Mapping Preserving the Kernel or the Range of Operators *

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Presented by Mostafa Mbekhta Received April 6, 2009

Abstract: Let X and Y be two infinite dimensional real or complex Banach spaces. In this note we determine the forms of surjective additive maps $\phi : \mathcal{L}(X) \to \mathcal{L}(Y)$ preserving the kernel's dimension or the range's codimension. As consequence, we establish that $\phi : \mathcal{L}(X) \to \mathcal{L}(X)$ preserves the kernel (respectively, the range) if and only if there exists an invertible operator $A \in \mathcal{L}(X)$ such that $\phi(T) = AT$ (respectively, $\phi(T) = TA$) for all $T \in \mathcal{L}(X)$.

Key words: Additives preservers, kernel operator, range operator.

AMS *Subject Class.* (2000): 47B48, 47A10, 46H05.

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* This note were written while the author was visiting the University of Lille1. He would like to thank all the members of mathematic's U.F.R for their beneficial hospitality.

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