EXTRACTA MATHEMATICAE Vol. **25**, Núm. 2, 151 – 164 (2010)

Improved Bounds in the Scaled Enflo Type Inequality for Banach Spaces *

Ohad Giladi, Assaf Naor

Courant Institute, New York University, New York, USA
giladi@cims.nyu.edu , naor@cims.nyu.edu

Presented by William B. Johnson Received November 14, 2010

Abstract: It is shown that if $(X, \|\cdot\|_X)$ is a Banach space with Rademacher type $p \geqslant 1$ then for every $n \in \mathbb{N}$ there exists an even integer $m \lesssim n^{2-1/p} \log n$ such that for every $f : \mathbb{Z}_m^n \to X$,

$$\mathbb{E}_{x,\varepsilon}\left[\left\|f\left(x+\frac{m}{2}\varepsilon\right)-f(x)\right\|_X^p\right] \lesssim_X m^p \sum_{j=1}^n \mathbb{E}_x\Big[\|f(x+e_j)-f(x)\|_X^p\Big],$$

where the expectation is with respect to uniformly chosen $x \in \mathbb{Z}_m^n$ and $\varepsilon \in \{-1,1\}^n$. This improves a bounds of $m \lesssim n^{3-2/p}$ that was obtained in [7]. The proof is based on an augmentation of the "smoothing and approximation" scheme, which was implicit in [7].

Key words: Rademacher type, metric characterization.

AMS *Subject Class.* (2000): 46B07, 46B20, 51F99.

References

[1] K. Ball, Markov chains, Riesz transforms and Lipschitz maps, *Geom. Funct. Anal.* **2** (2) (1992), 137 – 172.

[2] Y. Benyamini, J. Lindenstrauss, " Geometric Nonlinear Functional Analysis " Vol. 1, American Mathematical Society Colloquium Publications, 48, American Mathematical Society, Providence, RI, 2000.

[3] J. Bourgain, V. Milman, H. Wolfson, On type of metric spaces, *Trans. Amer. Math. Soc.* **294** (1) (1986), 295 – 317.

[4] P. Enflo, On infinite-dimensional topological groups, in " Séminaire sur la Géométrie des Espaces de Banach (1977û1978) ", Exp. No. 10-11, 11 pp., École Polytech., Palaiseau, 1978.

[5] O. Giladi, M. Mendel, A. Naor, Improved bounds in the metric cotype inequality for Banach spaces, *J. Funct. Anal.* **260** (1) (2011), 164 – 194.

*O. Giladi was partially supported by NSF grant CCF-0635078. A. Naor was supported in part by NSF grants CCF-0635078 and CCF-0832795, BSF grant 2006009, and the Packard Foundation.

[6] M. Gromov, Filling Riemannian manifolds, *J. Differential Geom.* **18** (1) (1983), 1 – 147.

[7] M. Mendel, A. Naor, Scaled Enflo type is equivalent to Rademacher type, *Bull. Lond. Math. Soc.* **39** (3) (2007), 493 – 498.

[8] M. Mendel, A. Naor, Metric cotype, *Ann. of Math. (2)* **168** (1) (2008), 247 – 298.

[9] A. Naor, G. Schechtman, Remarks on non linear type and PisierÆs inequality, *J. Reine Angew. Math.* **552** (2002), 213 – 236.

[10] G. Pisier, Probabilistic methods in the geometry of Banach spaces, in "Probability and analysis (Varenna, 1985)", Lecture Notes in Math., 1206, Springer, Berlin, 1986, 167 – 241.

[11] M. Ribe, On uniformly homeomorphic normed spaces, *Ark. Mat.* **14** (2) (1976), 237 – 244.