

Multiplicative Semigroups of Lipschitz Functions*

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Abstract: Given a (complete) metric space X , we denote by $\text{Lip}(X)$ the space of real-valued Lipschitz functions on X and we equip it with the pointwise product. The purpose of this note is to describe those bijections $T : \text{Lip}(Y) \rightarrow \text{Lip}(X)$ which are “multiplicative” in the sense that whenever $f, g \in \text{Lip}(Y)$ are such that $fg \in \text{Lip}(Y)$ one has $T(fg) = T(f)T(g)$.

The main result of the paper states that if X has no isolated points, then every multiplicative bijection $T : \text{Lip}(Y) \rightarrow \text{Lip}(X)$ arises as $T(f) = f \circ \tau$, where $\tau : X \rightarrow Y$ is a Lipschitz homeomorphism and so it is automatically linear.

We also give a description of the semigroup isomorphisms $T : \text{Lip}(Y) \rightarrow \text{Lip}(X)$ in the case where the underlying metric spaces are compact.

Key words: Semigroups of Lipschitz functions, homomorphism, representation.

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