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Topologies Corresponding to Continuous Representability of Preorders

Gianni Bosi

Università di Trieste, Dipartimento di Scienze Economiche, Aziendali, Matematiche e Statistiche "Bruno de Finetti", Piazzale Europa 1, 34127 *Trieste, Italy*
giannibo@econ.units.it

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Abstract: A topology τ on a fixed nonempty set X is said to satisfy the weakly continuous representation property if every weakly continuous not necessarily total preorder $\precsim$ on the topological space (X, τ) admits a continuous order preserving function. Such a property generalizes the well known continuous representation property of a topology τ on a set X (according to which every continuous total preorder $\precsim$ on (X, τ) admits a continuous order preserving function). In this paper I present some results concerning the topologies which satisfy the weakly continuous representation property.

Key words: Weakly continuous preorder, weakly continuous representation property, continuous representation property.

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