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Several Ways to a Berwald Manifold – and Some Steps Beyond *

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Abstract: After summarizing some necessary preliminaries and tools, including Berwald derivative and Lie derivative in pull-back formalism, we present several equivalent conditions, each of which characterizes Berwald manifolds among Finsler manifolds. These range from Berwald's classical definition to the existence of a torsion-free covariant derivative on the base manifold compatible with the Finsler function, the vanishing of the h-Berwald differential of the Cartan tensor and Aikou's characterization of Berwald manifolds. Finally, we study some implications of V. Matveev's observation according to which quadratic convexity may be omitted from the definition of a Berwald manifold. These include, among others, a generalization of Z.I. Szabó's well-known metrization theorem, and also lead to a natural generalization of Berwald manifolds, to *Berwald – Matveev manifolds*.

Key words: Berwald manifold, Ehresmann connection, parallel translation, averaged metric construction, Loewner ellipsoid.

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