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Derivations of Locally Convex $*$-Algebras

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Abstract: This is a survey account on some old and recent results on derivations from a locally convex ($*$-) algebra $A[\tau]$ in a locally convex A-bimodule. Essential differences with known results in the normed setting will be pointed out and natural questions will be put.

Key words: Locally convex algebra, pro-C^*-algebra, GB^*-algebra, locally convex bimodule, derivation, inner derivation.

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This paper is dedicated to the memory of Susanne Dierolf

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