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On Sequentially Right Banach Spaces

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Abstract: In this paper, we study the recently introduced class of sequentially Right Banach spaces. We introduce a stronger property (RD) and compare these two properties with other well-known isomorphic properties of Banach spaces such as property (V) or the Dieudonné property. In particular, we show that there is a sequentially Right Banach space without property (V). This answers a question of A.M. Peralta, I. Villanueva, J.D.M. Wright and K. Ylinen. We also generalize a result of A. Pełczyński and prove that every sequentially Right Banach space has weakly sequentially complete dual. Finally, it is shown that if K is a scattered compact Hausdorff space then the space $C(K, X)$ of X-valued continuous functions on K is sequentially Right (resp. has property (RD)) if and only if X has the same property.

Key words: Weakly compact operator, Right topology, Dunford-Pettis property, Property (V), Dieudonné property.

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