

Some Topological Invariants and Biorthogonal Systems in Banach Spaces

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Abstract: We consider topological invariants on compact spaces related to the sizes of discrete subspaces (spread), densities of subspaces, Lindelöf degree of subspaces, irredundant families of clopen sets and others and look at the following associations between compact topological spaces and Banach spaces: a compact K induces a Banach space $C(K)$ of real valued continuous functions on K with the supremum norm; a Banach space X induces a compact space B_{X^*} , the dual ball with the weak* topology. We inquire on how topological invariants on K and B_{X^*} are linked to the sizes of biorthogonal systems and their versions in $C(K)$ and X respectively. We gather folkloric facts and survey recent results like that of Abad-Lopez and Todorčević that it is consistent that there is a Banach space X without uncountable biorthogonal systems such that the spread of B_{X^*} is uncountable or that of Brech and Koszmider that it is consistent that there is a compact space where spread of K^2 is countable but $C(K)$ has uncountable biorthogonal systems.

Key words: cardinal invariants, biorthogonal systems, weak star topology, irredundance.

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