Some Problems Concerning Basic Sequences in Banach Spaces

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Abstract: In this note we present some open problems concerning the existence of certain sequences in a Banach space, we discuss known techniques and constructions related, and possible ways to attack them.

Key words: unconditional basic sequences, Ramsey methods, non-separable Banach spaces, separable quotient problem.

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References

[1] D. Amir, J. Lindenstrauss, The structure of weakly compact sets in Banach spaces, *Ann. of Math. (2)* **88** (1968), 35 – 46.

[2] S.A. Argyros, J. Lopez-Abad, S. Todorcevic, A class of Banach spaces with few non-strictly singular operators, *J. Funct. Anal.* **222** (2) (2005), 306 – 384.

[3] S.A. Argyros, A. Tolias, Methods in the theory of hereditarily indecomposable Banach spaces, *Mem. Amer. Math. Soc.* **170** (2004), no. 806.

[4] S.F. Bellenot, Tsirelson superspaces and ℓ_p, *J. Funct. Anal.* **69** (2) (1986), 207 – 228.

[5] J. Bourgain, F. Delbaen, A class of special $\mathcal{L}_\infty$ spaces, *Acta Math.* **145** (3-4) (1980), 155 – 176.

[6] J. Bourgain, "New Classes of $\mathcal{L}^p$-spaces", Lecture Notes in Mathematics, 889, Springer-Verlag, Berlin-New York, 1981.

[7] C. Brech, P. Koszmider, Thin-very tall compact scattered spaces which are hereditarily separable, *Trans. Amer. Math. Soc.* **363** (2011), 501 – 519.

[8] J. Bernues, I. Deliyanni, Families of finite subsets of $\mathbb{N}$ of low complexity and Tsirelson type spaces, *Math. Nachr.* **222** (2001), 15 – 29.

[9] S. Dilworth, E. Odell, Th. Schlumprecht, A. Zsak, Partial unconditionality, *Houston J. Math.* **35** (4) (2009), 1251 – 1311.

[10] P. DODOS, J. LOPEZ-ABAD, S. TODORCEVIC, Unconditional basic sequences in spaces of large density, *Adv. Math.* **226** (2011), 3297–3308.

[11] J. ELTON, "Weakly Null Normalized Sequences in Banach Spaces", Ph.D. Thesis, Yale University, 1978.

[12] T. FIGIEL, W.B. JOHNSON, A uniformly convex Banach space which contains no ℓ_p, *Compositio Math.* **29** (1974), 179–190.

[13] G. GODEFROY, On the diameter of the Banach-Mazur set, *Czechoslovak Math. J.* **60** (1) (2010), 95–100.

[14] W.T. GOWERS, Lipschitz functions on classical spaces, *European J. Combin.* **13** (1992), 141–151.

[15] W.T. GOWERS, B. MAUREY, The unconditional basic sequence problem, *J. Amer. Math. Soc.* **6** (1993), 851–874.

[16] A.S. GRANERO, M. JIMÉNEZ, A. MONTESINOS, J.P. MORENO, A. PLICHKO, On the Kunen-Shelah properties in Banach spaces, *Studia Math.* **157** (29) (2003), 97–120.

[17] C.W. HENSON, J. IOVINO, A.S. KECHRIS, E. ODELL, "Analysis and Logic", London Mathematical Society Lecture Note Series, 262, Cambridge University Press, Cambridge, 2002.

[18] W.B. JOHNSON, A reflexive Banach space which is not sufficiently Euclidean, *Studia Math.* **55** (1976), 201–205.

[19] W.B. JOHNSON, E. ODELL, The diameter of the isomorphism class of a Banach space, *Ann. of Math. (2)* **162** (2005), 423–437.

[20] W.B. JOHNSON, H.P. ROSENTHAL, On ω^*-basic sequences and their applications to the study of Banach spaces, *Studia Math.* **43** (1972), 77–92.

[21] J. KETONEN, Banach spaces and large cardinals, *Fund. Math.* **81** (1974), 291–303.

[22] J. LINDENSTRAUSS, L. TZAFRIRI, "Classical Banach Spaces I. Sequence Spaces", Ergebnisse der Mathematik und ihrer Grenzgebiete, Vol. 92, Springer-Verlag, Berlin-New York, 1977.

[23] J. LOPEZ-ABAD, S. TODORCEVIC, Partial unconditionality of weakly null sequences, *Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat.* **100** (1-2) (2006), 237–277.

[24] J. LOPEZ-ABAD, S. TODORCEVIC, Generic Banach spaces and generic simplexes, *J. Funct. Anal.* **261** (2) (2011), 300–386.

[25] J. LOPEZ-ABAD, S. TODORCEVIC, Positional graphs and conditional structure of weakly null sequences, preprint. http://arxiv.org/abs/1111.5150

[26] J. LOPEZ-ABAD, S. TODORCEVIC, Capturing families and finite dimensional approximations of Banach spaces, in preparation.

[27] B. MAUREY, H.P. ROSENTHAL, Normalized weakly null sequence with no unconditional subsequence, *Studia Math.* **61** (1) (1977), 77–98.

[28] S. NEGREPONTIS, Banach spaces and topology, in "Handbook of Set-Theoretic Topology", North-Holland, Amsterdam, 1984, 1045–1142.

[29] E. ODELL, A nonseparable Banach space not containing a subsymmetric basic sequence, *Israel J. Math.* **52** (1985), 97–109.

[30] E. ODELL, On Schreier unconditional sequences, *Contemp. Math.* **144** (1993), 197–201.

[31] E. ODELL, TH. SCHLUMPRECHT, The distortion problem, *Acta Mathe.* **173** (1994), 259–281.

[32] E. ODELL, TH. SCHLUMPRECHT, Distortion and asymptotic structure, in "Handbook of the Geometry of Banach Spaces", Vol. 2, North-Holland, Amsterdam, 2003, 1333–1360.

[33] TH. SCHLUMPRECHT, An arbitrarily distortable Banach space, *Israel J. Math.* **76** (1991), 81–95.

[34] A. SERSOURI, ω-independence in nonseparable Banach spaces, *Contemp. Math.* **85** (1989), 509–512.

[35] S. SHELAH, Uncountable constructions for B.A., e.g. groups and Banach spaces, *Israel J. Math.* **51** (1985), 273–297.

[36] S. TODORCEVIC, "Partition Problems in Topology", Contemporary Mathematics, 84, American Mathematical Society, Providence, RI, 1989.

[37] S. TODORCEVIC, Biorthogonal systems and quotient spaces via Baire category methods, *Math. Ann.* **335** (2006), 687–715.

[38] S. TODORCEVIC, "Walks on Ordinals and Their Characteristics", Progress in Mathematics, 263, Birkhäuser Verlag, Basel, 2007.

[39] B.S. TSIRELSON, Not every Banach space contains ℓ_p or c_0, *Funct. Anal. Appl.* **8** (1974), 138–141.