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Displaying Polish Groups on Separable Banach Spaces

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Abstract: A display of a topological group G on a Banach space X is a topological isomorphism of G with the isometry group $\mathrm{Isom}(X, \|\!|\cdot|\!\|)$ for some equivalent norm $\|\!|\cdot|\!\|$ on X, where the latter group is equipped with the strong operator topology. Displays of Polish groups on separable real spaces are studied. It is proved that any closed subgroup of the infinite symmetric group S_∞ containing a non-trivial central involution admits a display on any of the classical spaces c_0, $C([0,1])$, ℓ_p and L_p for $1 \leqslant p < \infty$. Also, for any Polish group G, there exists a separable space X on which $\{-1,1\} \times G$ has a display.

Key words: Polish groups, isometries of Banach spaces, linear representations, renormings of Banach spaces.

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