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Remarks on Gurariĭ Spaces

Joanna Garbulińska*, Wiesław Kubiś †

*Institute of Mathematics, Jan Kochanowski University,
Świetokrzyska 15, 25-406 Kielce, Poland; and
Institute of Mathematics, Faculty of Mathematics and Computer Science,
Jagiellonian University, Łojasiewicza 6, 30-348 Kraków, Poland

†Institute of Mathematics, Academy of Sciences of the Czech Republic,
Žitná 25, 115 67 Praha 1, Czech Republic, kubis@math.cas.cz

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Abstract: We present selected known results and some new observations, involving Gurariĭ spaces. A Banach space is *Gurariĭ* if it has certain natural extension property for almost isometric embeddings of finite-dimensional spaces. Deleting the word "almost", we get the notion of a *strong Gurariĭ* space. There exists a unique (up to isometry) separable Gurariĭ space, however strong Gurariĭ spaces cannot be separable. The structure of the class of non-separable Gurariĭ spaces seems to be not very well understood. We discuss some of their properties and state some open questions. In particular, we characterize non-separable Gurariĭ spaces in terms of skeletons of separable subspaces, we construct a non-separable Gurariĭ space with a projectional resolution of the identity and we show that no strong Gurariĭ space can be weakly Lindelöf determined.

Key words: Gurariĭ space, (almost) linear isometry, universal disposition, projection, rotund renorming, complementation.

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References

[1] P. Alexandroff, P. Urysohn, "Mémoire sur les Espaces Topologiques Compacts", Verhandelingen Amsterdam 14 (1929).

[2] A. Avilés, C. Brech, A Boolean algebra and a Banach space obtained by push-out iteration, *Topology Appl.* **158** (13) (2011), 1534 – 1550.

[3] A. Avilés, F. Cabello, J. Castillo, M. González, Y. Moreno, Banach spaces of universal disposition, *J. Funct. Anal.* **261** (9) (2011), 2347 – 2361.

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[4] R. Deville, G. Godefroy, V. Zizler, "Smoothness and Renormings in Banach Spaces", Pitman Monographs and Surveys in Pure and Applied Math., **64**, Longman Scientific & Technical, Harlow 1993.

[5] M. Fabian, "Gâteaux Differentiability of Convex Functions and Topology. Weak Asplund Spaces", Canadian Mathematical Society Series of Monographs and Advanced Texts, John Wiley & Sons Inc., New York, 1997.

[6] Ju.L. Gevorkjan, Universality of the spaces of almost universal placement (in Russian), *Funkcional. Anal. i Priložen* **8** (2) (1974), 72.

[7] V.I. Gurariĭ, Spaces of universal placement, isotropic spaces and a problem of Mazur on rotations of Banach spaces (in Russian), *Sibirsk. Mat. Ž.* **7** (1966), 1002 – 1013.

[8] V.I. Gurariĭ, Bases in spaces of continuous functions on compacta and certain geometric questions (in Russian), *Izv. Akad. Nauk SSSR Ser. Mat.* **30** (1966), 289 – 306.

[9] B. Jónsson, Homogeneous universal relational systems, *Math. Scand.* **8** (1960), 137 – 142.

[10] J. Kąkol, W. Kubiś, M. López-Pellicer, "Descriptive Topology in Selected Topics of Functional Analysis", Developments in Mathematics, Vol. 24, Springer Science+Business Media, New York, 2011.

[11] W. Kubiś, Fraïssé sequences: category-theoretic approach to universal homogeneous structures, preprint.

[12] W. Kubiś, S. Solecki, A short proof of isometric uniqueness of the Gurariĭ space, to appear in *Israel J. Math.*

[13] W. Kubiś, Linearly ordered compacta and Banach spaces with a projectional resolution of the identity, *Topology Appl.* **154** (3) (2007), 749 – 757.

[14] A.J. Lazar, J. Lindenstrauss, Banach spaces whose duals are L_1 spaces and their representing matrices, *Acta Math.* **126** (1971), 165 – 193.

[15] A.J. Lazar, J. Lindenstrauss, On Banach spaces whose duals are L_1 spaces *Israel J. Math.* **4** (1966), 205 – 207.

[16] J. Lopez-Abad, S. Todorcevic, Generic Banach spaces and generic simplexes, *J. Funct. Anal.* **261** (2) (2011), 300 – 386.

[17] W. Lusky, The Gurarij spaces are unique, *Arch. Math. (Basel)* **27** (6) (1976), 627 – 635.

[18] E. Michael, A. Pełczyński, Separable Banach spaces which admit l_n^∞ approximations, *Israel J. Math.* **4** (1966), 189 – 198.

[19] A. Sobczyk, Projection of the space (m) on its subspace (c_0), *Bull. Amer. Math. Soc.* **47** (1941), 938 – 947.

[20] P. Wojtaszczyk, Some remarks on the Gurarij space, *Studia Math.* **41** (1972), 207 – 210.

[21] M. Zippin, Extension of bounded linear operators, in "Handbook of the Geometry of Banach Spaces, Vol. 2", (Edited by W.B. Johnson and J. Lindenstrauss), North Holland, Amsterdam, 2003, 1703 – 1741.

[22] V. Zizler, Locally uniformly rotund renorming and decompositions of Banach spaces, *Bull. Austral. Math. Soc.* **29** (2) (1984), 259 – 265.