EXTRACTA MATHEMATICAE Vol. **27**, Núm. 1, 91 – 123 (2012)

Odd Jacobi Manifolds: General Theory and Applications to Generalised Lie Algebroids

Andrew James Bruce

Pembrokeshire College, Haverfordwest, Pembrokeshire, SA61 1SZ, UK
andrewjamesbruce@googlemail.com

Presented by Janusz Grabowski Received February 1, 2012

Abstract: In this paper we define a Grassmann odd analogue of Jacobi structure on a supermanifold. The basic properties are explored. The construction of odd Jacobi manifolds is then used to reexamine the notion of a Jacobi algebroid. It is shown that Jacobi algebroids can be understood in terms of a kind of *curved* Q-manifold, which we will refer to as a quasi Q-manifold.

Key words: supermanifolds, Jacobi structures, Lie algebroids, Q-manifolds.

AMS *Subject Class.* (2010): 17B70, 53D10, 53D17, 58A50, 83C47.

References

[1] M. Alexandrov, M. Kontsevich, A. Schwarz, O. Zaboronsky, The Geometry of the master equation and topological quantum field theory, *Internat. J. Modern Phys. A* **12** (1997), 1405 – 1429.

[2] P. Antunes, C. Laurent-Gengoux, Jacobi structures in supergeometric formalism, *J. Geom. Phys.* **61** (2011), 2254 – 2266.

[3] V.I. Arnold, "Mathematical Methods of Classical Mechanics", Translated from the 1974 Russian original by K. Vogtmann and A. Weinstein (corrected reprint of the second edition 1989), Graduate Texts in Mathematics, 60, Springer-Verlag, New York.

[4] I.A. Batalin, G.A. Vilkovisky, Gauge algebra and quantization, *Phys. Lett. B* **102** (1981), 27 – 31.

[5] I.A. Batalin, G.A. Vilkovisky, Quantization of gauge theories with linearly dependent generators, *Phys. Rev. D (3)* **28** (1983), 2567 – 2582.

[6] Ö.F. Dayi, Odd time formulation of the Batalin-Vilkovisky method of quantization, Modern Phys. Lett. A **4** (1989) 361 – 365.

[7] A. Coste, P. Dazord, A. Weinstein, "Groupoïdes Symplectiques", Publ. Dép. Math. Nouvelle Sér. A, 87-2, Univ. Claude-Bernard, Lyon, 1987.

[8] J. Grabowski, G. Marmo, Jacobi structures revisited, *J. Phys. A* **34** (2001), 10975 – 10990.

[9] J. Grabowski, G. Marmo, The graded Jacobi algebras and (co)homology, *J. Phys. A* **36** (2003), 161 – 181.

[10] J. Grabowski, P. Urbański, Algebroids–general differential calculi on vector bundles, *J. Geom. Phys.* **31** (1999), 111 – 141.

[11] J. Grabowski, Graded contact manifolds and principal Courant algebroids, `arXiv:1112.0759 [math.DG]`, 2011.

[12] J. Grabowski, M. Rotkiewicz, Higher vector bundles and multi-graded symplectic manifolds, *J. Geom. Phys.* **59** (2009), 1285 – 1305.

[13] D. Iglesias, J.C. Marrero, Generalized Lie bialgebroids and Jacobi structures, *J. Geom. Phys.* **40** (2001), 176 – 199.

[14] H.M. Khudaverdian, Semidensities on odd symplectic supermanifold, *Comm. Math. Phys.* **247** (2004), 353 – 390.

[15] T. Lada, J. Stasheff, Introduction to SH Lie algebras for physicists, *Internat. J. Theoret. Phys.* **32** (1993), 1087 – 1103.

[16] M. de León, J.C. Marrero, D. Martín de Diego, Linear almost Poisson structures and Hamilton-Jacobi equation, Applications to nonholonomic mechanics, *J. Geom. Mech.* **2** (2010), 159 – 198.

[17] A. Lichnerowicz, Les variétés de Poisson et leurs algèbres de Lie associées, *J. Differential Geometry* **12** (1977), 253 – 300.

[18] S.L. Lyakhovich, A.A. Sharapov, Characteristic classes of gauge systems, *Nuclear Phys. B* **703** (2004), 419 – 453.

[19] S.L. Lyakhovich, E.A. Mosman, A.A. Sharapov, Characteristic classes of Q-manifolds: Classification and applications, *J. Geom. Phys.* **60** (2010), 729 – 759.

[20] F. Petalidou, On a new relation between Jacobi and homogeneous Poisson manifolds, *J. Phys. A* **35** (2002), 2505 – 2518.

[21] A.S. Schwarz, Geometry of Batalin-Vilkovisky quantization, *Comm. Math. Phys.* **155** (1993), 249 – 260.

[22] R.A. Mehta, Differential graded contact geometry and Jacobi structures, `arXiv:1111.4705v1 [math.SG]`, 2011.

[23] D. Roytenberg, On the structure of graded symplectic supermanifolds and Courant algebroids, in "Quantization, Poisson Brackets and Beyond" (Manchester, 2001), Contemp. Math., 315, Amer. Math. Soc., Providence, RI, 2002, 169 – 185.

[24] A.Yu. Vaĭntrob, Lie algebroids and homological vector fields (Russian), *Uspekhi Mat. Nauk* **52** (1997), 161 – 162; translation in *Russian Math. Surveys* **52** (1997), 428 – 429.

[25] Th. Voronov, Graded manifolds and Drinfeld doubles for Lie bialgebroids, in "Quantization, Poisson Brackets and Beyond" (Manchester, 2001), Contemp. Math., 315, Amer. Math. Soc., Providence, RI, 2002, 131 – 168.

[26] Th.Th. Voronov, Q-manifolds and higher analogs of Lie algebroids, in "XXIX Workshop on Geometric Methods in Physics", AIP Conf. Proc., 1307, Amer. Inst. Phys., Melville, NY, 2010, 191 – 202; also avaliable as `arXiv:1010.2503v1 [math.DG]`