Approximation by Polynomials in a Weighted Space of Infinitely Differentiable Functions with an Application to Hypercyclicity

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Abstract: A space of infinitely differentiable functions defined on an open cone of $\mathbb{R}^n$ and of prescribed growth near the boundary of the cone and at infinity is considered. The problem of polynomial approximation in this space is studied. It is shown that every linear continuous operator on this space that commutes with each partial derivative operator and is not a scalar multiple of the identity is hypercyclic.

Key words: Hypercyclic operators, polynomial approximation.

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