EXTRACTA MATHEMATICAE Vol. **27**, Núm. 1, 31 – 58 (2012)

Cyclicity Results for Some Antianalytic Toeplitz Operators Acting on H^p

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Presented by Alfonso Montes Received October 27, 2010

Abstract: This article deals with some cyclic families of functions for antianalytic Toeplitz operators whose symbol is a finite Blaschke product in the spaces H^p where $1 < p < \infty$. We give a description of the invariant subspaces for this type of operators generated by special decompositions and by lacunary decompositions of functions. To that end, we study some particular decomposition properties of a function in H^p associated with an inner function which are valid in general.

Key words: Cyclicity, Hardy spaces, Toepliz operators, Blaschke products.

AMS *Subject Class.* (2010): 47A16, 47B35, 47B38, 30H10, 47A15.

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