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Some Results on Automatic Continuity of Group Representations and Morphisms

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Abstract: In the first part of the paper, some criteria of continuity of representations from a locally compact group in a Banach algebra are given. The common feature of these results is the fact that the continuity of a representation can be deduced from the continuity of its composition with some linear forms on its range. The second part uses the result of the first part to deduce automatic continuity results of Haar-measurable morphism from locally compact groups to infinite dimensional linear or unitary groups.

Key words: Automatic continuity, locally compact groups, linear representations, Glicksberg-De Leeuw decomposition.

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