Superstability of Approximate Cosine Type Functions on the Monoid $\mathbb{R}^2$

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Abstract: In this paper, we study the superstability problem for the cosine type functional equation

$$f(x_1 x_2, x_1 y_2 + x_2 y_1) + f(x_1 x_2, y_1 x_2 - x_1 y_2) = 2 f(x_1, y_1) f(x_2, y_2)$$

on the commutative monoid $(\mathbb{R}^2, \times)$. As a result we obtain cosine type functions satisfying the equation approximately.

Key words: Functional equation, cosine function, superstability, multiplicative function.

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