

## On Dilation of Local Semigroups of Contractions and some Applications\*

RAMÓN BRUZUAL, MARISELA DOMÍNGUEZ, ÁNGEL PADILLA

*Escuela de Matemática, Fac. Ciencias, Universidad Central de Venezuela,  
Apartado Postal 47686, Caracas 1041-A, Venezuela  
ramonbruzual.ucv@gmail.com, ramon.bruzual@ciens.ucv.ve*

*Escuela de Matemática, Fac. Ciencias, Universidad Central de Venezuela,  
Apartado Postal 47159, Caracas 1041-A, Venezuela  
marisela.dominguez@ciens.ucv.ve, dominguez.math@gmail.com*

*Escuela de Matemática, Fac. Ciencias, Universidad Central de Venezuela  
angel.padilla@ciens.ucv.ve*

Presented by Alfonso Montes

Received August 5, 2012

*Abstract:* We show that a multiplicative family of contractions on a separable Hilbert space, with parameter on the interval  $[0, 1)$  of the dyadic rationals, has a unitary dilation with parameter on the dyadic rationals and values on a larger Hilbert space. This result is used to prove a dilation result for strongly continuous local semigroups of contractions. As an application we give results of extension of positive definite functions on the line, generalizing the Kreĭn extension theorem.

*Key words:* Operator semigroup, contraction operator, isometric operator, positive definite.

*AMS Subject Class. (2010):* 47D03, 43A35, 42A82

### REFERENCES

- [1] R. BRUZUAL, Local semigroups of contractions and some applications to Fourier representation theorems, *Integr. Equ. Oper. Theory* **10** (1987), 780–801.
- [2] R. BRUZUAL, M. DOMÍNGUEZ, Extensions of operator valued positive definite functions and commutant lifting on ordered groups, *J. Funct. Anal.* **185** (2) (2001), 456–473.
- [3] R. BRUZUAL, M. DOMÍNGUEZ, M. MONTILLA, On unitary dilations of two-parameter semigroups of contractions and continuous commutant lifting, *Acta Sci. Math.* **77** (3-4) (2011), 607–620.
- [4] M.L. GORBACHUCK, Representation of positive definite operator functions, *Ukrainian Math. J.* **17** (1965), 29–46.
- [5] M. GROSSMAN, H. LANGER, Über indexerhaltende Erweiterungen eines hermiteschen operators in Pontrjaginraum, *Math. Nachr.* **64** (1974), 289–317.

---

\*All authors were supported in part by the CDCH of the Univ. Central de Venezuela.

- [6] M.G. KREĬN , Sur le problème du prolongement des fonctions hermitiennes positives et continues, *Dokl. Akad. Nauk. SSSR* **26** (1940), 17–22.
- [7] M.A. NEUMARK, Spectral functions of a symmetric operator, *Bull. Acad. Sci. URSS, Sér. Math.* **4** (1940), 277–318.
- [8] M. PTAK, Unitary dilations of multiparameter semigroups of operators, *Ann. Polon. Math.* **45** (1985), 237–243.
- [9] Z. SASVÁRI, “Positive Definite and Definitizable Functions”, Akademie Verlag, Berlin, 1994.
- [10] B. SZ.-NAGY, C. FOIAS, “Harmonic Analysis of Operators on Hilbert Space”, North Holland Publishing Co., Amsterdam-London, 1970.