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Derivations of Generalized B*-algebras

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Abstract: It is well known that a commutative C*-algebra has no nonzero derivations. In this article, we extend this result to complete commutative GB*-algebras having jointly continuous multiplication. We also give some results about derivations of GB*-algebras, with their underlying C*-algebras being W*-algebras.

Key words: GB*-algebra, topological algebra, derivation, locally convex bimodule.

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