

Transfer Operators on Complex Hyperbolic Spaces

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Abstract: Let \mathbb{B}^n be the unit ball in the n -dimensional complex space and let Δ be the Bergman Laplacian on it. For $\lambda \in \mathbb{C}$ such that $|\Re(i\lambda)| < n$ we give explicitly the transfer operator from the space of holomorphic functions \mathbb{B}^n onto an eigenspace $E_\lambda^+(\mathbb{B}^n)$ of Δ . This answers a question raised by Eymard in [2]. As application, for $\lambda = -i\eta$ with $0 < \eta < n$, we get that the classical Hardy space $H^2(\mathbb{B}^n)$ is isometrically isomorphic to the space

$$H_\lambda^2(\mathbb{B}^n) = \left\{ F \in E_\eta^+(\mathbb{B}^n) : \sup_{0 < r < 1} (1 - r^2)^{\frac{n-\eta}{2}} \left[\int_{\partial\mathbb{B}^n} |F(r\theta)|^2 d\theta \right]^{\frac{1}{2}} < \infty \right\}.$$

Consequently $H_\lambda^2(\mathbb{B}^n)$ is a Banach space.

Key words: Transfer operator, Hardy spaces.

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